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Journal of Teacher Education 2011 62: 8
DOI: 10.1177/0022487110378851

Journal of Teacher Education
62(1) 8–22

DOI: 10.1177/0022487110378851
http://jte.sagepub.com

Learning to Teach in the Figured World of Reform Mathematics: Negotiating New Models of Identity

Jasmine Y. Ma[1] and Marcy Singer-Gabella[1]

Abstract

Starting from the assertion that traditional and reform mathematics pedagogy constitute two distinct figured worlds of teaching and learning, the authors explore the initiation of prospective teachers into the figured world of reform mathematics pedagogy. To become successful teachers in reform-oriented classrooms, prospective teachers must learn more than pedagogical tools and moves: They must understand what it is to participate in the figured world of reform pedagogy, develop models of identities for participants in this world, and negotiate new constructions of mathematics. In this article the authors present three episodes from an elementary mathematics teacher education class where positions of "teacher" and "child" were offered by instructors in activities designed to approximate practice in the reform figured world. Students negotiated new models of identity and conceptions of mathematics as they took up these positions in varying ways.

Keywords

teacher education, mathematics pedagogy, models of identity, figured worlds, reform pedagogy

Learning to engage in reform mathematics pedagogy entails developing a vision of teaching and learning mathematics that differs dramatically from the vision underlying traditional classroom models (Feiman-Nemser, 2001). Although traditional, or didactic, mathematics teaching is characterized by a routine of presenting a procedure, modeling an example problem, and then asking children to practice similar problems (Stigler & Hiebert, 1999), reform mathematics pedagogy entails designing and posing tasks that call on children to reason about quantities, invent their own strategies, and discuss their thinking (e.g., Boaler & Humphreys, 2005). These contrasting routines in turn require distinct forms of participation for all those involved. In the reform version, children must be supported in, and held accountable for, making mathematical sense of problems and communicating their reasoning, not just for producing a correct answer. Problems are not merely contexts for practicing mathematical skills and procedures but are instead sites of meaningful mathematical activity. Teachers design and facilitate this activity, leveraging students' reasoning to inform next steps in instruction and to advance learning (cf. Lampert, 1990, 2001).

In this article, we pursue the question of how teacher candidates might come to make sense of the vision and routines of reform mathematics pedagogy. Adapting analytic tools from scholarship in anthropology and sociolinguistics, we examine candidates' changing conceptions of what it means to do, learn, and teach mathematics as reflected in their "models of identity" for learners and teachers. Our analysis is based on the study of a mathematics foundations course for prospective elementary teachers over the course of one semester. In the course, new and existing models of identity became visible as instructors positioned teacher candidates as either learners or prospective teachers in the figured world of reform mathematics and as candidates took up or resisted those positionings in various ways.

Analytic Framework

We find it useful to view the two approaches to mathematics described above as constituting distinct "figured worlds" (Holland, Lachicotte, Skinner, & Cain, 1998). As defined by Dorothy Holland and her colleagues (1998), a figured world is "a socially and culturally constructed realm of interpretation in which particular characters and actors are recognized, significance is assigned to certain acts, and particular outcomes are valued over others" (p. 52). Artifacts and signs are attributed meaning that might differ from how those outside of the figured world interpret them. People, actors in the figured world, have expectations for how events unfold and how others will behave in these events. The figured worlds of both

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traditional mathematics pedagogy and reform pedagogy are peopled with children, teachers, parents, other faculty in the school, administrators, curriculum developers, and so forth; however, the similarities end there. Although the general outcome of "learning" is valued in both worlds, the specifics of what constitutes learning and its indicators differ. The responsibilities and relationships between teachers and children differ as well, as do the significance of problem solving and the routines of classroom activity.

Boaler and Greeno (2000) provide apt illustration of these worlds in the context of high school mathematics teaching and learning. In their analysis of interviews with high school calculus students, the researchers identify two distinct figured worlds of mathematics, one driven by didactic teaching and the other by discussion-based teaching. These two worlds closely resemble those that we characterize as "traditional" and "reform." The didactic teaching world is highly ritualized and features procedures presented by teachers, with students practicing these procedures alone. Students are receivers of knowledge (procedures) and charged with reproducing it (through problem solving). In the discussion-based world, students had opportunities to discuss problems with their peers and as a class. They were active learners, making sense of knowledge (relationships between quantities and mathematical ideas) together. These two different worlds had implications not only for how students learned but also for their perceptions of math and for how they identified themselves as mathematics learners and doers.

Other educational researchers have drawn on the figured worlds perspective to study the relationship between prospective teachers' learning and their identities. Horn, Nolen, Ward, and Campbell (2008) found that students' emerging identities as teachers served as a resource for "editing" the figured worlds of teacher education coursework and practice teaching into a learning context that made sense and was useful to them. They define identity as "the way a person understands and views himself, and is often viewed by others, at least in certain situations" (p. 62; cf. Cain, as cited in Lave & Wenger, 1991, p. 81; Holland et al., 1998, p. 68). This definition, although not representative of all perspectives,[1] has a feature common to most sociocultural approaches in the educational research literature on identity: an emphasis on context and recognition. More specifically, within a particular context, others recognize an identity as it is being enacted or animated.

Recent research on the development of preservice mathematics teacher identity is consistent with this conception, highlighting the relationships among prospective teachers' identities as learners, doers, and teachers of mathematics and the contexts and practices in which they are situated. For example, starting from the premise that teacher identities are profoundly shaped by the normalizing practices of schools as institutions, Walshaw (2004) examined the ways in which preservice teachers' identities were constituted and "consented to" (p. 65) in the context of primary schools during their field placements. She reports that for many preservice teachers "this new space [the field] was fraught with ambiguous and sometimes painful negotiations to produce individual subjectivity" (p. 78).

In another study, Nicol and Crespo (2003) found that preservice teachers' images of themselves as mathematics teachers were integrally related also to their identities as learners and knowers of mathematics. In providing continuing opportunities for peer collaboration and critique as students developed case studies of their practice, the course Nicol and Crespo studied helped students develop identities as learners who "make sense of mathematics, pursue multiple solutions to problems, and make connections within and beyond the discipline" (p. 378). In turn, students began to aspire to become teachers who could promote such activity in their classrooms, continually learning about mathematics and mathematics teaching in and from practice.

Writing from a social justice perspective, de Freitas (2008) sought to challenge her secondary mathematics teacher education students' identities as knowers of mathematics. If preservice teachers can come to recognize that their identities as successful mathematics students are socially constructed, she argued, they might "become more responsive to their students' diverse identities." She explains, "Conceiving of identity as socially constructed and confined by classroom discourse can help teachers trouble the dominant 'mastery' identity associated with success in school mathematics" (p. 44).

In each of these studies, prospective teachers' identities as learners, doers, and novice teachers of mathematics are understood to be dynamically constituted within particular contexts, malleable, and intertwined. We take up this view, and seek to address an aspect of preservice teacher identity development relatively unexplored in teacher education literature. Although a number of teacher education researchers have studied the development of prospective teachers' identities in relation to particular forms and contexts of practice, few have allotted attention to the ways in which learners, or newcomers to communities of practice, come to recognize different identities in the first place. An exception is the research reported by Ronfeldt and Grossman (2008). Informed by the work of Hazel Markus, the authors use the concept of "possible selves" to explore the professional identity development of students preparing to become clergy, clinical psychologists, and teachers. They consider the ways in which professional education might enable students to encounter, try on, and practice these possible selves to support the development of their own professional identities.

In Ronfeldt and Grossman's (2008) framework, "As people observe others in the professional role, they generate a repertoire of potential identities" (p. 42). In this article, we consider how students might generate this repertoire if their access to professionals, and the world as a whole, is limited. In addition, we argue that it is not just the potential identities for *teachers* that are important but also those for children learning mathematics. Stanton Wortham's (2006, 2008) idea of *models of identity* provides a way to reference the many possible identities recognized in a given figured world. Wortham (2006) describes models of identity as accounts of "what some people are like" (p. 6); in other words, a constrained repertoire of behavior that some people have for certain circumstances and the reasons for their behavior. For example, in a traditional mathematics classroom, a "good math teacher" might be thought of as one who is patient and gives good explanations of procedures. If a student is having trouble with long division, the teacher might offer a more detailed or different explanation for each step of the process and walk the student through more example problems. This would make sense in a traditional mathematics class where students' trouble could be interpreted as not being able to remember the steps correctly or misinterpreting how or when to carry each step out.

Models of identity are sociohistorically developed and circulated but negotiated in local settings. As Wortham (2006) illustrates in his research, local models of identity are constrained by the widely circulating equivalents but also are shaped by the particular events of the class's experiences together, such as activities and discussions. In the high school classroom he studied, the teacher and female students drew on widely circulating models for boys and girls in school to develop "an intensive local metapragmatic model of promising girls and unpromising boys" (p. 43). Unpromising boys were understood by members of the class (although not necessarily beyond it) to be resistant to classroom expectations, less intelligent, and unlikely to succeed (p. 53). This model of identity was then further developed in relation to a particular male student in the class.

In this article, we extend prior work on the development of prospective teachers' identities as learners and teachers of math by examining how models of identity are surfaced and negotiated early in candidates' pedagogy programs. In the class we present here, instructors faced the challenge of teaching students about new models of identity that fit in a reform world of mathematics against a backdrop of more widely circulating traditional models. Together, instructors and students negotiated their taken-as-shared models of identity for elementary school learners and teachers of mathematics by surfacing and testing their (often implicit) accounts of what learners and teachers of math are like. At stake was the ability both to recognize reform models of identity and to interact with them. As described below, because there were no elementary school children and teachers learning math "in the room," the negotiation occurred through role-play or projection.

The reader will note that each of our examples of models of identity references specific subject matter. Following Hull and Greeno (2006), Boaler and Greeno (2000), and others, we contend that in educational settings, practice and therefore identity are shaped in relation to the subject matter of learning. Jackson (2007, 2009) emphasizes that the construction of subject matter in a classroom frames and is framed both by what mathematics is learned and by the social identities of learners. She gives the example of a fifth grade classroom in which mathematics was constructed through classroom practices and discourse as a set of procedural tasks where speed and right answers were valued. This construction of mathematics was informed by institutional discourses about the student population of the school and framed the learning, participation, and mathematical socialization and identification of individual students in the class. We further argue that in teacher education courses, there are strong ties between the construction of mathematics and the construction and negotiation of existing models of identities for teachers and children learning mathematics, not just for the social identities of students themselves.

Teacher education courses that value reform pedagogy, then, must do more than teach different forms of knowledge and strategies pertinent to the technical craft. They must make students familiar with and participants in the figured world, both reshaping their various models of identity of children and teachers and introducing or promoting an appropriate construction of mathematics. Since the regular activity of teacher education courses typically takes place away from actual classrooms with teachers and children, all this must happen without direct access to that figured world and the different enacted identities that people it. How then can instructors of these courses make this world visible and available to students?

In the class we describe here, the instructors represented the figured world of reform pedagogy with the support of approximations of practice (Grossman et al., 2009) that required students to take on roles as teachers or learners. For example, Lewis, the lead instructor of the class, might ask students to solve an addition problem without using the traditional algorithm and to be able to explain their solutions in terms of quantity. He was clear that he was both modeling a way of teaching and teaching them math content at the same time. This was an example of what we considered to be asking the students to take on the role of a child learning mathematics. These activities and tasks were intended to give students a glimpse of both reform teaching and a child's perspective on mathematical content.

It turned out that, in practice, students often did not maintain these roles and at times did not even take them up. Although it would be possible to characterize this simply as resistance, we find it more productive to look more closely at these episodes as dynamically unfolding, jointly produced interactive events (cf. Garfinkel, 1967). We ask not why students failed to enact predefined roles but instead how they were positioned (by instructors, themselves, and each other) and how they understood and did or did not take up the content of these positions. Davies and Harré (1990) describe positioning in contrast to the idea of a preexisting cultural structure where a role entails a predefined script. Instead, agentic participants take up positions in dynamic and subjective ways. Speakers position themselves or others by invoking ways of being and interacting in particular kinds of situations. Embedded in such positionings are both the relations between participants and their assumptions and understandings of how one plays the "character" in which one is positioned. Participants who have been positioned may choose to take up the position or to reposition themselves. Whichever path they take, their own histories, their current interpretations of the interaction, and how they see themselves within the interaction have consequences for their response.

In the context of teacher training, for both instructors and researchers, positioning can be a pedagogical tool (placing students as participants in an approximation of the figured world—such that relevant features are salient to the students) as well as an assessment tool (Do they recognize the salient features? How do they respond to them? How do they enact particular models or take up particular positions?). As teacher education students are positioned and position themselves in this figured world, they enact and test their models of identity for learners and teachers of mathematics. It is through the dynamic process of positioning that these models of identity are negotiated and revised.

In this article we analyze three episodes from the course: two where students were positioned as children learning mathematics and a more extended one where students were positioned as teachers. We argue that the instructors offered the reform versions of these positions through their talk, the tasks in which they engaged students, and the local history shared by the class. Students took up (or refused) these positions in various ways, usually drawing from models of identity composed of elements from both the traditional and reform mathematics worlds.

Method

The research presented here is part of an ongoing longitudinal study of prospective teachers' developing understandings of learning in mathematics and science, the enactment of these understandings as they begin teaching, and the learning outcomes of their students. The objectives of the larger project are to build models of teacher learning trajectories that might inform the design of teacher preparation programs and to support the development of appropriate tools to assess the impact of teacher preparation program features on elementary and middle school student learning. In the second and third years of the project, the research team conducted intensive observations of pedagogy courses taught at the project's two participating institutions: videotaping and taking field notes on class sessions, gathering artifacts of instructional practice (e.g., planning documents, task descriptions), and collecting evidence of prospective teachers' learning as reflected in work samples from major assignments and in some cases through interviews.[2]

Context of Study

This article follows a course taught at one of the project sites that is designed to help prospective teachers begin to unpack their understandings of mathematics content and to develop new conceptions of what it might look like to know, learn, and teach mathematics. The course, Mathematics for Elementary Teachers, is located within an undergraduate elementary education program that aims, in line with reform mathematics values, to prepare teachers to make visible and to analyze student reasoning about content as both pedagogical method and basis for planning next instructional steps. Most students take this course early on, following two broad survey courses—one on issues in education, the other on exceptionality taught in the Department of Special Education. Thus, this is usually the first opportunity for students to consider in-depth questions of teaching and learning with regard to a particular subject area and age level. During the semester we observed, the course instructors, Lewis and Susan, were doctoral students who both had worked for several years with a prominent mathematics researcher and teacher educator. Like past instructors of this course (as well as those of other, similar courses at the university), they conjecture that prospective teachers come to their classes socialized into a construction of mathematics that includes a highly constrained view of mathematical reasoning and, in turn, an undifferentiated sense of what it might mean for a child's knowledge of mathematics to develop. In particular, prospective teachers tend to think about children's mathematics largely in terms of whether a child gets a problem right or wrong (Interview, May 18, 2008).

To disrupt this right–wrong binary, instructors have sought to help students deconstruct their own understandings of mathematics content and what it means to learn and teach mathematics. Therefore, course sessions are devoted to engaging teacher candidates in mathematics ideas and reflecting on the forms of pedagogy that might support students in developing their understandings of this content. Faculty have built into these courses a sequence of tasks designed to help prospective teachers understand that (a) mathematical knowledge develops in the relation between action and mental representation, (b) children's solution strategies provide insight into their mathematical knowledge, and (c) teachers can make productive use of these solution strategies to help children develop more advanced understandings.

A typical class session might involve discussing the previous night's readings or reporting back on a homework assignment and then spending the bulk of the time working through and discussing mathematical tasks. The tasks could include a problem like "Yesterday, Timothy ate 1/4 of his chocolate bar. Today he ate 2/3 of what was left. How much did he eat today?" or an activity like "The division operation 17/5 might have the solution 3 R2, 3, 4, or 3 2/5. Think of a problem that might lead to each of those solutions." Discussions focused on students' understandings of the mathematics involved in problem solving and children's possible solution strategies, among other topics. Over the course of one of these problem-solving and activity sessions, students were continuously (re)positioned, by themselves and other students as well as by the instructors of the course. Thus, as described below, through this instructional routine, instructors modeled high frequency, high leverage practices in mathematics teaching and also provided a context for students to deepen their mathematical content knowledge.[3]

Participants

In the spring of 2008, the class consisted of two instructors and 11 female students, 10 of whom were undergraduate sophomores and juniors preparing to teach in elementary classrooms.

Data Sources

Our analysis is based on observations, field notes, and video of 18 of the 28 class sessions that occurred between January and April 2008. We also collected corresponding classroom artifacts, including handouts and course assignments. In addition, we conducted several informal interviews with the instructors over the course of the semester.

Data Analysis

In our broader study, we are concerned with the development of prospective teachers' understandings of mathematics concepts and pedagogy in the domain of number and numeric algorithm. Therefore, in the current analysis we selected the 18 sessions emphasizing this content area. Because the broader study in part aims to answer questions about how instructors' design assumptions play out in the development of prospective teachers' knowledge and pedagogy, we began by partitioning sessions into episodes according to instructional activities. Thus, each episode featured a single instructional task or cluster of related tasks designed to support the learning goals for the given session. Episodes were typically 25 to 35 minutes in length (sessions were 75 minutes). Each episode was transcribed, and then the video and corresponding transcript were imported into a video software program (Transana) to enable successive rounds of coding.

We chose for focused analysis one episode from each of six different class sessions spread across the semester: two from the beginning, two from the middle, and two toward the end. We chose these episodes for their diversity of content, looking for some that were routine as well as some in which trouble occurred (according to the instructors). Based on our observations of course sessions and our preliminary examination of the range of episodes, we determined that six episodes selected from six sessions would provide sufficient representation of classroom events while remaining a manageable set for fine-grained analysis.

As we reviewed the episode video and transcripts and began to notice the different ways in which students were positioned, we created, then refined, categories for these positions and focused on different configurations of the data to begin to theorize about their import. Broadly, we observed students taking on positions as university students of pedagogy, as adult learners of mathematics, as teachers of elementary mathematics ("teacher"), as well as elementary school children learning math ("child"). We did not treat these positions as mutually exclusive, understanding that multiple figured worlds were always at play. Instead, we coded for a position if there was any sign that the speaker was taking it up.

Members of the research team took independent passes through the target episodes before we met together to negotiate positioning categories (see the appendix for coding guidelines). Once we reached agreement on both the definitions of the categories and our coding of the episodes in relation to these categories, we grouped together excerpts illustrating each position to consider how the positions were taken up. We analyzed interactions for the resources students used to take up or reject positions. In particular, we focused on the models of identity students and instructors seemed to be drawing from and if or how they shifted over the course of the semester. We also coded and analyzed episodes in sessions occurring immediately before or after the target episodes to see if models of identity seemed to be locally stable or idiosyncratic. We triangulated observation notes and video with student assignments, work samples, and interviews (formal and informal) with instructors.

The following sections present excerpts from three of the target episodes to illustrate our analysis of the ways in which candidates drew on and negotiated emerging models of identity for young learners and teachers of mathematics as they were variously positioned, at different times throughout the semester. We present these excerpts in particular as relatively typical with respect to the variety of ways that the students took up and negotiated positions.

Findings

Models of Identity for Children Learning Mathematics

When posing problems, the instructors usually asked students to solve without using a traditional algorithm, to find strategies that children might come up with, and to think of as many strategies as possible. At the start of each problem, and then in debriefing their work, students were asked implicitly to imagine themselves as children learning mathematical content in the context of a reform-oriented mathematics class. Although students experienced this new way of being students of mathematics, their mathematical content knowledge was also often problematized. Meanwhile, the instructors modeled the kinds of teaching that they advocate, occasionally making comments about their pedagogical decisions and moves.

Episode 1. Students chose to take up or refuse this "child" positioning in many different ways. During the second week of class, Lewis asked the class to solve a subtraction problem:[4]

> T/L: I would like for you to solve this problem today in as many ways as you can come up with. I will give you a few minutes to think about it. You can talk with other people if you would like and then we will look at some of the methods by which you've solved the problem. A book has 64 pages, you've read 37 of those pages, how many pages do you have left to read? Be sure that for any method you use that you can explain how you did it in terms of quantity of pages. Come up with as many ways of solving it as you can. [TR 01/23/2008][5]

This task does not explicitly position the class as children. However, in class sessions leading up to this one, Lewis has talked about his vision of good instruction, which includes starting with a problem with a context, soliciting student strategies, and comparing them (ON 01/16/2008, 01/21/2008). He has also emphasized that children should understand that they are working with quantities and not just manipulating symbols according to seemingly arbitrary rules. Given this shared history, it is likely that many students in the class interpreted this task as an example of one a reform-oriented math teacher would pose to children. Indeed, in this episode and in similar ones occurring throughout the semester, the students respond in ways that confirm this interpretation. A reasonable response to this reading of the activity for the students would be to "play the part" of those children, which they do in different ways, drawing from various models of identity for children learning mathematics.

The students worked on the problem and talked to each other about it while the instructors circulated to check on their progress. After several minutes, Lewis asked four of the students to put their methods on the board. Rosie was asked to share her solution first:

> R: I wrote out tally marks for all 64 pages of the book and counted 37 of them and that's how many you have already counted, you've already read, so the remaining number of tallies is how many pages are left. And that's 27.

Rosie seems to have taken up the child position willingly, finding a successful strategy and describing it in terms of quantities of pages as Lewis requested. Bridgit, who shared her solution next, demonstrated a method whereby she added 10 to the original 37 until she reached 67. This meant she had added 30; however, the book had only 64 pages, so she had to subtract three from the 30 to find the actual number of pages she had left.

Others in the class, however, had more difficulty with the task. After the class discussed the two solutions, Amy expressed the trouble that she and Jody had coming up with solution strategies:

> A: Well it is kind of like what Jody and I like keep saying to each other. We keep trying to think of problems that like a kid is going to know how to do, like a first grader is not, I don't think, like capable of like thinking about like, like one of my problems is 37 plus blank, like you could do like an algebraic kind of problem, but I feel like these are problems that they would never do because they're not even going to even understand how to do it.

Amy's comment that they "keep trying to think of problems that like a kid is going to know how to do" supports our suspicions that these two students, at least, tried to take on the role of "child" in this activity. That no one asked them with incredulity why they were doing such a thing provides further evidence. However, they had trouble doing so. Amy explains that she (re)represented the original problem as an "algebraic kind of problem," "37 plus blank." Although this representation was intended as a possible solution strategy, Amy realized she had transformed the problem into one from a more advanced type of traditional school math (algebra)—one unlikely to be accessible to a first grader.

One interpretation of Amy and Jody's difficulties in coming up with solution strategies to the original problem is that their thinking was entirely constrained by their school mathematics knowledge, and so they could find ways of solving the problem only in those terms. Another is that they have misunderstood the activity and tried to rewrite the problem instead of solve it. A third interpretation—from a figured worlds and models of identity perspective—is that Amy and Jody do not have enough experience with reform mathematics settings to envision how children might respond to such a task. Lewis gives Amy the opportunity to say more about their trouble coming up with methods:

> T/L: Are you suggesting that first graders could not solve this problem?
>
> A: No, like I think they could, but like the ways that we have come up with, I think are a little far-fetched and that is why I have such a hard time coming up with them because I am thinking on like a first grade basis like pie charts, like something that like

you know like, I feel like they wouldn't be able to ever figure any of these out on their own.

T/L: Something I could have brought today was the actual results of kids in I think first grade or second grade maybe in the Netherlands, I should have brought that. I will bring it maybe next time. It was a whole bunch of different strategies that kids came up with and some of them were the exact things that you guys came up with today. So in fact=

A: Cause I feel like if they did come up with this, then they would probably be able to solve 64 minus 37.

Lewis asks Amy if she thinks "that first graders could not solve this problem." Amy explains that she thinks that the solutions the class has come up with are "a little far-fetched." The model of identity for children, first graders in this case, that Amy draws from is that they work with "pie charts" and "they wouldn't be able to ever figure any of these out on their own," or "if they did come up with this, then they would probably be able to solve 64 minus 37." In her talk, Amy constructs mathematics as cumulative and having a set sequence. Implied is that the important mathematics in the problem Lewis presented was 64 minus 37, and the goal of doing it was to learn subtraction. However, the solutions that Rosie and Bridgit offered do not seem to count as "solving" for Amy, and in fact they were treated as more difficult than just "solving." For Amy and Jody, first graders have a limited amount of mathematical knowledge that includes pie charts but not thinking algebraically. They also have a standard trajectory of development in which solving a subtraction problem would or should come before some of these more complicated strategies.

This model of identity is informed by and has consequences for their figuring of the world of mathematics teaching and learning. In other sessions, Amy and Jody identified themselves as students who came from (and as undergraduates, are still taking classes in) a traditional math education setting. Their experiences had mostly been in figured worlds where procedures were taught first and emphasized and conceptual understanding became peripheral. The enduring construction of mathematics was one emphasizing procedural skill and correct answers over reasoning with quantities. Children who did not keep up were seen as "not good at math." This kind of understanding of math education framed Amy and Jody's model of identity for children learning mathematics. In this excerpt from early in the semester, they drew from this model of identity when trying to imagine (or enact) participating in a math classroom as a child.

Episode 2. Even as the instructors sought to establish an alternative, and new worlds and models began to emerge for the students, this figured world of traditional mathematics was elaborated alongside the reform alternative. In the class session preceding the one referenced above, Lewis startled students by showing that he could make the traditional algorithm for solving addition and subtraction problems "work" from any direction. After he demonstrated that one could calculate from left to right as well as right to left, Jamie argued that "you can't start from the middle because you'd have to borrow from both sides." Amid bursts of laughter and surprise from the students, Lewis took up the challenge:

T/L: ((*writes on board*))

$$\begin{array}{r} 4\;2\;5 \\ -\,1\;6\;7 \\ \hline \end{array}$$

I'm going to start in the middle. ((*uncertain laughter from students*)) Um, six from two can't do that, so I have to do that, *((“borrows” ten 10s from the 100s place, writing 12 instead of 2 in the tens place*)), that's six. Um, should I go to the left or the right now?

$$\begin{array}{r} {}^{3}\cancel{4}\;{}^{1}2\;5 \\ -1\;\;6\;7 \\ \hline 6\;\; \end{array}$$

Ja: (3s) I don't know. ((*students giggle*))

T/L: I'll go to the left. ((*smiling*)) Three minus one, that's two ((*writes 2 to the left of the 6 on the bottom line)).* Now I'll go to the right. Five from seven::n, can't do that, I'll have to take one. ((*crosses out the 6 in the bottom line, and writes 5; changes the 5 to a 15. Some students chuckle.*))

$$\begin{array}{r} {}^{3}\cancel{4}\;{}^{1}2\;{}^{1}5 \\ -1\;\;6\;\;7 \\ \hline 2\;\;\cancel{6}\;\;8 \\ 5\;\;\;\; \end{array}$$

Ja: Okay so y- but you're just DOing like the same way of carrying that WE do, tha- do your left version ((*referring to earlier example of moving from left to right*)). Like, you didn't do it from the BOttom like, like you were doing before, you carried like the way that most kids in the U.S. are taught to do. You carried from the top.

T/L: Oh. Well I didn't HAVE anything at the bottom yet.

Ja: Yeah. So you- kind of, it's like– both versions= ((*smiling*))

T/L: I cheated? (1s) ((*students laugh*))

Ja: I don't know. I feel like that's a little confusing. ((*laughs*)) [TR 01/21/2008]

In calling out the fact that Lewis "carried [from the top] . . . the way that most kids in the U.S. are taught to do" it rather than "from the bottom," Jamie expresses the expectation that a "starting from the middle" algorithm should somehow be totally different than the traditional algorithm—doing anything similar, or using both versions is cheating (a sentiment wryly reflected back by Lewis). Somehow, Lewis has misappropriated a procedure from the figured world of traditional mathematics into what remains for Jamie an ill-defined figured world of reform mathematics, and she finds this move confusing. Note that Jamie accuses Lewis of using "the same way of carrying as WE do," distancing herself from whatever group of people it is that is associated with starting from the middle. She takes ownership of both the traditional algorithm and the world in which it resides, separating them from Lewis's new world.

Stephanie finds Lewis's manipulation of the algorithm problematic for a different reason:

S: If you have um, some kids may get confused as like, you put an ANswer where the ANswer should GO? But it's not the answer that it's going to be? Does that make sense? ((*another student is saying yeah*)) Like you put the six there where the answer would be, but that's not going to BE part of the answer. Which I think may be confusing for SOME kids.

As the uncomfortable laughter that accompanied Lewis's work at the board reflects, in students' figured worlds of traditional mathematics, there is a proper place for each digit at each step of the routine. Disrupting these conventions—or even appropriating them for an unconventional procedure—creates confusion: "You put the six there where the answer would be, but that's not going to be part of the answer . . . which I think may be confusing for some kids." In Stephanie's model of identity for children, young learners have expectations for which numerals belong where in a mathematical procedure that looks like that and violating those expectations could lead to confusion. Entangled with this model of identity is the way that mathematics is constructed for these children. Mathematics has answers (final answers, not intermediate ones) that are to be found in particular physical locations within universally known procedures. (Note also the assumption that this algorithm, presented by Lewis as a way to problematize the traditional algorithm as "the way to do it," was something that might be presented to children.)

As the exchange continued, Lewis pushed back at Stephanie's assertion that children would have defined a "right place" to put the answer:

T/L: Why? When will kids know that there's where the answer goes? Why- what makes [kids think the ans-
A: [As soon as you tell them-
T/L: As soon as you tell them. ((*students laugh*)) [What if you don't tell them that?
A: [it goes below the line. Yeah.
Jo: It gives them a nice sense of consistency and it's something they can always COUNT on.
T/L: [Does it give-
Jo: [That's why algorithms are COMfortable. ((*students laugh*))
T/L: Does it- does it- provide THEM comfort or does it provide YOU comfort, as a teacher?
A: That's the THING though, like, what if you started teaching like a new like- bizarre method like that to your students and then they went home to get help from their parents and the parents are like what in the world are you doing? ((*students and T/S begin talking all at once*))

Here, Lewis explicitly juxtaposes the model of identity for teachers of math underlying the students' responses against an alternative. For the students, teaching mathematics involves telling children how to use the algorithm. At the same time, in asking what would happen if the teacher did not tell children where the answer goes, he surfaces their model of identity for children learning math. After being taught solution methods by the teacher ("What if you started teaching like a new like- bizarre method like that?"), children go home and need help from their parents. The model of identity for children learning math again involves struggling with procedures rather than reasoning about mathematical problems. Lewis hints here to the students that mathematics is socially constructed in the classroom. He tells them that there is not a correct place for answers to go until teachers tell children that there is and that mathematics could be constructed differently if teachers did not do this, thus heading off the confusion.

Interestingly, this figured world of traditional mathematics is populated not only by teachers and students but also by parents who have similar expectations for how mathematics will be taught and learned. The students here are concerned with children's and parents' confusion. They worry that children will be confused if the traditional algorithm is mixed with Lewis's "bizarre" one and if numbers that are not part of the answer show up where the answer belongs. Parents will be confused when faced with helping their children with "bizarre" methods.

In fact, the students were often concerned during this semester with the confusion of actors in their developing figured worlds of mathematics—children, their parents, teachers the children might have in future years, even administrators in the schools. As in the excerpts presented above, the expectation of confusion (along with students' own confusion) often resulted when students imposed or transposed models of identity (in whole or part) from one figured world on/to actors from another: interpreting signs of identity by drawing from conflicting models of identity or constructing a version of mathematics incompatible with the figured world in which an individual was positioning himself or herself.

Models of Identity for Teachers

Lewis and Susan occasionally presented activities that positioned the students as teachers of elementary mathematics, such as asking them to look at a child's work and diagnose his or her thinking. In addition, either while or after they shared strategies for solving problems, students were sometimes asked to imagine themselves as teachers. Lewis and Susan would ask questions such as, "What might a child be thinking if he used this strategy?" or "What might you ask a child next?" At other times, students reflected on activities in which they were positioned as children but asked questions about what a teacher might do in particular situations.

Episode 3. The following excerpt from February begins with students giving the instructors some feedback on how they feel the course is progressing:

> Ja: And I was gonna say that I feel like I understand that you want us to be able to think about how kids are thinking about this and diagnose where they're at? But I feel like what I DON'T know is- have- any idea of how to get them to the next step. So I feel like that's where, I feel like we're not being TAUGHT how to TEACH, in that way? Like you know, I don't know how to bring them- I feel like I understand like where they're at and what the steps are but I have no idea how to- make that change. Does that make sense?
>
> T/L: Hmm hmm. Okay. Here's, here's one for you. You feel like the kid, if he's going to add um or if he's gonna, COUNT things, like the kid has to have it in front of him at all ti- Like- you're like, "Man, I feel this kid always has to move the objects around the table." What would be your next step?
>
> Ja: I don't know. I know I'd WANT him to be able to do it by a counting strategy? But I don't know what in the WORLD I would say or do to make him get to that step. [TR 02/11/2008]

Jamie recognizes children's thinking as an important and consequential "text" in the figured world of reform mathematics. She has begun to envision a developmental sequence of strategies that children might bring to solving problems. She is also developing a model of identity where certain kinds of teachers "think about how kids are thinking" and "diagnose where they're at." However, she describes these teacher moves as ones "you want us to be able" to do rather than as activities that fit into her world of mathematics pedagogy. This model of identity is something that Lewis and Susan value and that she feels she understands, but not necessarily one that she accepts for herself. She continues by stating that she has not learned how to help a child progress, given a particular diagnosis. She says, "I feel like we're not being TAUGHT how to TEACH, in that way?" Jamie's emphasis on the word *teach* here, followed by *in that way*, indicates that she is articulating a definition of "teaching" in which considerations of student thinking or developmental trajectories are not constitutive—they are part of a different "way" of teaching. At this point in the term for Jamie, the significant part of teaching (which she states she is not being taught) is "mak[ing] a change" in children's thinking, getting them from their current "step" to the next one and eventually to an end goal of some sort. Lewis proposes a scenario in which a child counts with direct modeling of physical objects and asks the members of the class what they would do as teachers. He positions them as teachers here, asking them to place themselves in this situation with this particular child whom they know is a direct modeler and to imagine what they would do next. This model of teacher identity is clearly situated in the reform world, engaged in diagnosing student thinking. Jamie responds that, although she knows she'd "WANT [the child] to do it by a counting strategy," she does not know "what in the WORLD I would say or do to make him get to that step." Here, we researchers might want to ask to which world Jamie refers. One interpretation is that she is referring to her current understanding of the reform world, and the very point of her criticism of the course so far is that the instructors have yet to "teach" her what one can do or say in this world to make a direct modeler get to a counting strategy (after reading of a draft of this article, Lewis shared this interpretation of the episode with us). Note the parallels between Jamie's figuring of teaching elementary mathematics and the teaching she seeks from Lewis and Susan. Jamie makes a clear request for her instructors to tell her the right procedure for getting the student to the next step; however, this is not a move made in the figured world of reform mathematics represented by Lewis and Susan nor in the figured world of mathematics teacher education enacted in their university classroom. Another interpretation is that Jamie has searched in the figured world of mathematics pedagogy of which she *is* knowledgeable and does not know which possible thing that she could "do or say" is the right one.

Our argument is not whether Jamie has sufficient knowledge of a collection of pedagogical moves. If she does, her interpretations of them would be very different given her construction of mathematics and the figured world in which she locates herself. Through her talk here and in earlier episodes, we see that, in Jamie's figured world, the act of teaching continues to be defined narrowly in terms of the transmission of skill or knowledge: diagnosing children's thinking and using that thinking to support next steps in learning lie beyond (but not contrary to) this definition. She can see the value in diagnosing children's thinking but does not yet think of it as a way to leverage their learning. The word *teach* is constituted by *mak[ing]* the child get to the next step. In this world, children have little agency or ownership in their learning.

In contrast, in the figured world of reform mathematics depicted by Lewis, student thinking and reasoning are valued not just as formative assessment but as a mechanism by which a child will learn. Jamie's use of the word *make* to describe the teacher's role in advancing a child's knowledge frames teaching moves as determinate, as if there is a set list of actions a teacher needs to implement in a given situation to "make" learning happen. In Jamie's world of mathematics pedagogy, teachers move children through a series of steps, or stages of mathematical understanding, by carrying out prescribed moves which children will respond to and learn from. At the end of this section we contrast this to Lewis's version of advancing a child's knowledge. Although Jamie rejected Lewis's positioning as not belonging in her world of mathematics pedagogy, Stephanie had a different response:

T/L: Any ideas?
S: I mean, could- I'm not sure if this is exactly answering your question, but if you HAVE the objects, could you say like, say you were doing "three plus three equals six." And you have three objects and three objects and you're trying to get the answer. You co::uld, say well write down how many are in this group in a like a formatted equation like with a plus sign and two- three blanks and an equal sign. So they write down the three and then the next three and then all together it's six. Does that make any sense?
T/L: That makes sense. I think that's a step beyond what Jamie or what I'M saying. You're, you're moving in towards some formal notation and really concentrating on, um the, the numerals involved.
T/S: And what would be your justification for wanting to do that Stephanie, based on the situation? What made you think that that would be what you'd want to do?
S: Okay. Beca::use, they see that the actual objects are there?
T/S: Hmm hmm.
S: So they- re- represent the number::r that- you're writing down? So that actual objects represent the numbers when you're doing math problems.
T/S: That's a really important, CONcept.
T/L: Hmm hmm. It is.

Stephanie takes up the position of teacher but hesitantly, beginning with the disclaimer, "I'm not sure if this is exactly answering your question." She distances herself a little from being a teacher by using *you* in her response instead of *I*, as Jamie did, although she could be responding directly to Jamie, suggesting what she could do since she had no ideas. Her suggestion focuses, as Lewis points out, on numerals and symbols. When Susan asks her to explain her reasoning, Stephanie talks about making the connection between quantities of objects and the symbols "when you're doing math problems." Her figured world of mathematics pedagogy is one in which it is important always to connect quantity to mathematical operations. So in progressing from direct modeling to writing addition equations, she would make sure that a child knew what the numbers represent.

The way Stephanie constructs mathematics here reflects aspects of a reform mathematics world as well as a traditional world. She privileges reasoning about quantities, but at the same time she talks of "doing math problems" as if they are always symbolic. What counts as a "math problem" has "numbers," and it is important for children to recognize that there are objects, or quantities, represented by these numbers. This emphasis on connecting quantity to notation and operation is certainly one that Lewis and Susan (and the reform mathematics community) value: Recall the example above where they required that students be able to explain their solutions in terms of quantities. However, Lewis interprets her response as getting too far ahead. He would want to make sure that the child learned something in between direct modeling and symbolic representation, which he later reveals to be mental representation.

T/S: Any ideas?
Jo: Can you repeat what, we're, trying, to do?
T/L: Mmm hmm. The kid, that I'm talking about always has to, have PHYsical objects to count them. If you wanted to do three plus three. They've got to put three things here on the table and three things here on the table and then count them. How do we get the kid aWAY from having to direct MOdel everything? What should I DO?
A: You said you think hers is maybe a step beyond what- you're- (looking for)?=
Jo: Well maybe they could do something like, in the videos we watched with like patterns, and dots, and trying to promote that like mental imagery, and even moving away from like, PHYsical tangibles to- I know fingers are still tangibles but I feel like it's closer and maybe moving the child towards, sort of putting it in their head? (I don't,) visualization=
T/L: I think you're right. [I think I would-
A: [(Imageries, and-)=
T/L: I think I would flash a picture, for ONE second. Three dots and three dots. And I'd say how many was that? And the kid would be FORCED, to n- NOT have stuff to count on the table. And it would put it in their head, this picture.

Susan gives students another chance to respond to Lewis's scenario. Jody asks for it to be repeated, and after Lewis gives another explanation, Amy verifies that Stephanie's "is maybe a step beyond what- you're- (looking for)?" She is hesitant to jump in with a suggestion and treats the scenario as a task with a correct answer, the one that Lewis is "looking for."

She does not take up the position of teacher but instead remains firmly seated in the classroom as a teacher education student.

Jody similarly treats the scenario as a task and explicitly draws from past activities in class ("like in the videos we watched"). She takes up the position in the sense that she speaks from the perspective of a teacher who has diagnosed a child as a direct modeler and who has the goal of helping the child progress in her learning—although she distances herself (and others in the room) from this teacher by using the pronoun "they." In negotiating her model of this teacher, she (appropriately) uses resources from earlier in the semester to reason about the appropriate next step for the child and the move the teacher might make.

Note that Jody's response draws not only from past class sessions but also from this conversation as it unfolded. Her use and emphasis of the word *physical* was a direct echo of the way that Lewis used it in his most recent, more elaborated description of the scenario. Like Amy, she heard that a focus on numerals was a step too far from what Lewis was thinking. Stephanie's concern with "answering [the] question" along with Lewis's response to her ("I think you're right . . .") could have made salient a classroom activity–like participation framework (Goffman, 1981), prompting Amy and Jody to both treat it more in that way. Therefore, her positioning as university student should not be taken as resistance of a teacher position but, rather, an additional layer of activity mediating the interaction.

At one level, Lewis's revoicing of Jamie's initial question, "How do we get the kid aWAY from having to direct MOdel everything?" and his phrase "the kid would be FORCED to n- NOT have stuff to count on the table" seem to convey a view of teaching incompatible with a world of reform mathematics. However, we propose that the two models of teacher identity and the figured worlds in which they operate are distinguished by the nature of the action and the underlying view of mathematics he describes. Jamie speaks of "getting [children] to the next step" and "mak[ing] that change" in the child's reasoning. In contrast, Lewis describes constraining the problem space so that a child will have to reason in a new way to find a new strategy for solving the problem. Although Lewis makes a conjecture about what the child's new strategy will be (mental imagery), he does not suggest transmitting that step to child. Rather, he proposes providing a new problem space in which the child can reason. Lewis's construction of mathematics is as a practice that involves reasoning with available resources. At this point, tools such as constraining the problem space are absent from Jamie's figured world of mathematics. Similarly, her models of teacher and child identity and their relationships to each other do not include things such as altering the problem space to leverage different ways of student thinking or reasoning with quantities in meaningful ways given available resources.

The preceding extended example depicts various ways in which the position of "teacher" was taken up and negotiated. Each response was contingent on individuals' developing figured worlds of mathematics pedagogy, corresponding constructions of mathematics, and relevant models of identity as well as their shared local history and the dynamically unfolding interaction. As illustrated in the last exchange, by midsemester these developing figured worlds reflected a kind of hybridization of the traditional and reform worlds, as students drew on old and new resources to envision the practice of teaching mathematics.

The figured world/models of identity lens affords a view of a student's complaint that he or she is not being taught "how to teach" (and other moments of confusion or breakdown) as active reasoning about how elements in an unfamiliar and unavailable reform figured world work. Jamie's question could be seen as a plea for additional course content and a gripe about current instructional routines. Similarly, Amy's assertion that first graders do not come up with certain kinds of strategies could be interpreted as a lack of knowledge about child development or skepticism about the value of the activity. Our analysis does not refute these interpretations. Rather, it allows us to look for the relationship between students' current understandings and instructional goals and tease out previously unacknowledged sources of trouble.

In addition to learning mathematical content and pedagogical knowledge, students are learning about teaching and learning that not only looks very different from the traditional version but also has very different values. We are reminded that how students make use of existing and newly learned pedagogical resources will be shaped and constrained by their conceptions of learners, teachers, and what it means to do, learn, and teach mathematics—not to mention those held by other stakeholders in the contexts in which they find themselves.

Discussion

Above, we have given examples of how students in a one-semester teacher education course were positioned by instructors as either teachers of or children learning mathematics. Lewis and Susan presented the figured world of reform pedagogy (along with relevant models of identity) in at least two ways over the course of the semester: by describing it and approximating it in class activities. They stressed the importance of posing problems that are mathematically meaningful and accessible to various levels of understanding so that students could reason with real quantities and share their solutions. They described the teacher's role in orchestrating student explanations so as to promote efficiency, accuracy, and flexibility. They enacted approximations of this world by posing such problems, modeling the teacher's moves, and prompting students to both come up with solutions and imagine next pedagogical steps. In their talk and activity, mathematics was constructed as a domain constituted by relationships among real quantities, multiple ways to reason about them, and strategies for problem solving that could be more or less sophisticated and efficient.

Although they often talked explicitly about some aspects of the figured world of reform mathematics, like what problem solving might look like in the classroom and valued ways of treating student reasoning, the instructors did not explicitly address corresponding models of identity. These were developed and negotiated indirectly as students were positioned as teachers and children. Students took up or rejected these positions in various, sometimes unexpected ways. Building on their past experiences, the shared local history of the class, and the current dynamically unfolding interaction, they drew from developing local models of identities for teachers and children, an evolving social construction of mathematics, and both traditional and reform figured worlds. Some students more readily entered into the world presented by their instructors, whereas others stood at the periphery, preferring to reposition themselves as university students of mathematics pedagogy. These students seemed more comfortable with explicit "metatalk" rather than participating in approximations of practice. Whether taking up positions as students of pedagogy, children learning mathematics, teachers, or some combination of these, students' models of identity for teachers and young learners of mathematics, as well as their conceptions of the practices of learning and doing math, were reflected in their talk and the questions they posed. Meanwhile, models of identity, constructions of mathematics, and students' understandings of the figured world of reform mathematics pedagogy continued to be negotiated.

Note that the models of identity addressed here are general: Our discussion has not taken into account the many variations of models for children and teachers within the figured world of reform mathematics that might circulate in a class. Participants in the figured world of either traditional or reform mathematical pedagogy may recognize more or less successful children, enthusiastic children, resistant children, and so on. By addressing only the general categories of "teachers" and "children" we did not intend to neglect more specific categories but instead to highlight the way the models were drawn from and developed and how they interacted with figured worlds and constructions of mathematics. It is an open question as to whether students were making distinctions between these variations or if, at this early stage of their teacher training, they were considering just "typical" versions.

The lens of figured worlds thus affords a view of the learning of individual students in relation to the class collective and the practice for which they are preparing. This view is especially important for understanding the preparation of reform-oriented teachers, whose observation, practicum, and employment experiences might very well be in more traditional settings. Although the disjuncture between reform-oriented methods course teachings and field-based demands are commonly conceived as a mismatch problem, the figured world lens pushes us to understand how individuals are interpreting their situations. Horn and her colleagues (2008) argue that the worlds of coursework and practice teaching offer different affordances and constraints for learning and that the students in their study ordered and edited these worlds to reflect their own goals and emerging identities. We build on their work by drawing attention to the many identities that populate a figured world. Students grapple with their own developing teacher identities, but these are in relation to the other people that populate the world of teaching, including students, their parents, other teachers, and administrators. Teasing out the salient models of identity (along with the construction of mathematics) that students use to reason about teaching and learning events can help researchers look past the apparent discontinuity between mathematics teacher education coursework and real-life classroom situations (Ward, 2009) to a more complex consideration of the ways prospective teachers interact within and interpret these contexts.

Mathematics teacher education research concerned with questions of emerging teacher identities often focuses on the time period when teacher candidates begin to engage in practices of teaching: during their field placements. This is consistent with the definition of identity as constituted in practice. Our research suggests that a model of identity perspective contributes a productive lens for examining the ways in which prospective teachers come to recognize, differentiate, and navigate teacher identities (and their corresponding figured worlds) during university-based coursework as they attempt more limited approximations of teaching practice. In attending to ways in which conflicting models and images of learning and teaching are taken up, acted on, and revised over time, analysts can follow learning in a course or over a longer period. Although we have not tracked the learning of individual students or of the class in this article, it is clear in our examples that there were shifts in the models of identity students drew on, the way they constructed mathematics in their talk, and their conceptions of the figured world of reform pedagogy. For example, at the beginning of the course, some students in the teacher education class that we studied agreed that first graders would not be able to come up with sophisticated strategies to solve a subtraction problem on their own. By midsemester, students had changed their minds and drew readily from a model of identity of children who reason in various ways about quantities. Stephanie's suggestion for how to help the direct modeler advance to a more sophisticated strategy took for granted that it was important for him to keep quantity in mind. Stephanie and Jody spontaneously took up resources from the figured world of reform mathematics to help them reason about how they might support this child's learning.

In addition, existing research in the mathematics teacher education literature tends to focus either on preservice teachers' mathematical identities (e.g., de Freitas, 2008; Nicol & Crespo, 2003) or their developing identities as teachers (e.g., Horn et al., 2008; Walshaw, 2004). A consideration of figured worlds of mathematical teaching and learning and models of identity necessarily has implications for how mathematics is socially constructed for teachers and children. This perspective can help researchers explore how preservice teachers' learning in their university-based coursework interacts with both their mathematical and teacher identities.

The consideration of models of identity within figured worlds may also be productive for teacher education practice. Many of the ways in which Lewis and Susan represented and approximated the figured world of reform mathematics in their course are not uncommon in teacher education courses. Although others have noted that positioning teachers as both students and teachers in mathematical activity can enable novices to build and practice a repertoire of pedagogical tools and moves (e.g., Ball & Forzani, 2009; Bowers & Doerr, 2001), the concern for models of identity also turns our attention to the ways in which instructors use these approximations of practice to help novices recognize and reason about their multiple and often conflicting expectations for learners and teachers in any given context. As illustrated above, these expectations shape and constrain how subject matter comes to be defined and enacted and how pedagogical resources are taken up. In Lewis's and Susan's course, especially in the first part of the semester, the new models of identity, constructions of mathematics, and figured world of reform mathematics were presented with the world of traditional mathematics pedagogy as an inevitable backdrop. As students mixed elements from the two worlds, confusion or conflict emerged. At times this confusion was productive, affording an opportunity for calling attention to new ways of framing the work of teaching or conceptions of mathematics and learning. At other times—and for some students in particular—confusion simply led to shutdown (which was confirmed in course evaluations and informal conversations with the instructors). We conjecture that teacher educators might work with the confusion, deliberately (and perhaps explicitly) leveraging the multiple models of identity students bring to university-based coursework to help them understand the people and practices of the figured worlds of mathematics that they will encounter in the field, and so more effectively manage and refine their repertoires. Such leveraging would require instructors to have a vision of how various models of identity may fruitfully be highlighted, juxtaposed, and elaborated in relation to particular forms and contents of practice.

Finally, we propose that although identity is often defined in terms of how one is seen by others, the act of seeing others as certain kinds of people is itself an enactment of a way of being in the world—a reflexive positioning in a figured world or community of practice. At the same time that available models of identity influence developing identities, developing identities shape individuals' understandings of different models of identity. In the episodes presented here, prospective teachers' models of identity for children informed and were informed by their conceptions of what it means not only to learn math but also to teach it. In recognizing certain models of identity for children learning math, students in Mathematics for Elementary Teachers drew on corresponding models of identity for elementary mathematics teachers. As instructors and students surface, address, and extend models of identity for children, they have an opportunity to expand the range of potential models of identity for teachers of mathematics as well.

Appendix

Positionings Coding Definitions

We are concerned with how students take up (or resist) the positions offered by instructors (and sometimes each other or themselves). Positions are offered by instructors as they pose tasks and respond to student contributions. Positions are offered or taken up by students in the ways they participate in tasks and in their talk when asking questions or making arguments.

1. The *University Students of Pedagogy* (USP) position refers to instances of metatalk about teaching and learning. This is explicit conversation about how students learn and what teachers do and when.
   - Instructors position students:
     - Instructors break in the middle of teaching and modeling with commentary about what they just said or how they just presented something
     - Instructors take an extended turn to talk about what they are about to do or what they just did as related to students' learning to become teachers or about children's learning and understanding of mathematics
     - Instructors refer to homework assignments and reading they've had
   - Students position themselves or each other:
     - Students ask a question about how something would be taught
     - Students refer to homework assignments they've had
2. The *Teacher* position refers to places where students are asked to consider what they would do given a particular student or classroom situation.
   - Instructors position students:
     - Looking at children's work and trying to understand their reasoning as well as talking about the next steps to advance a child in his or her thinking

    - Thinking about which numbers or wording is more appropriate for a particular task
    - Invoking a hypothetical classroom in talk to make an argument (in combination with USP)
  - Students position themselves or each other:
    - Thinking about which numbers or wording is more appropriate for a particular task
    - Invoking a hypothetical classroom in talk to make an argument (in combination with USP)
3. The *Child* position refers to instances where students are asked to consider or enact how children might think about a math problem or concept.
  - Instructors position students:
    - Ask them to solve a problem—without using the algorithm or adult conceptual resources, with as many strategies as possible, or as a child might solve it
    - Ask them to explain their solutions to each other, often in terms of quantity; they ask them questions comparing solutions or if one will always work
    - Have them use known children's strategies to solve problems
  - Students position themselves or each other:
    - Take the perspective of a child in talk to make an argument ("If I were a kid I would be confused" or "When I was a kid my teacher helped me by . . .")
4. The *Learner of Mathematics* position refers to places where the students are positioned as learning mathematical content or asked (implicitly) to reconsider aspects of the practice or discipline of mathematics. This might happen by itself or in combination with one of the other codes.
  - Instructors position students:
    - Pose tasks that require mathematical reasoning (including both giving problems to solve and asking why a solution works or is efficient)
  - Students position themselves or each other:
    - Ask about mathematical content

Declaration of Conflicting Interests

The author(s) declared no potential conflicts of interests with respect to the authorship and/or publication of this article.

Funding

The author(s) disclosed receipt of the following financial support for the research and/or authorship of this article: grant #ESI-0554486 from the National Science Foundation.

Notes

1. For example, see Gee (2000) and Sfard and Prusak (2005) for two very different viewpoints.
2. The two institutions were chosen for their common emphasis on teaching that is guided by strong knowledge of disciplinary content and student thinking. One of the participating institutions is a large (34,000 students) state-related university; the other (the site examined in this article) is a small (11,300 students) private university. Both are located in major urban centers with highly diverse populations.
3. Ball and Forzani (2009) assert that although the particulars of any given classroom may be unpredictable and context specific, in fact there are some very predictable routines in which teachers engage to support student learning in a given content area. Such routines or practices include posing and representing problems in math and leading whole-class discussion of a work of literature in English. Ball and Forzani characterize these routines as "high frequency" and advocate that they be explicitly taught during teacher preparation. The power of a routine (what makes it "high leverage") derives from its potential to advance students' learning of particular content *and* a teacher's insight into student learning. In contrast to routines designed to promote management efficiency, high leverage routines are construed as "activity structures that teachers [can] *think with*, rather than behaviors that become automated" (Kazemi, Lampert, & Ghousseini, 2007, p. 4). In our research, we are seeking evidence that the instructional routine described here was itself not only high frequency but also high leverage.
4. Our transcript convention is a modification of Gail Jefferson's method (Atkinson & Heritage, 2006). The sequential flow of interaction is privileged in our choice to place speakers' turns one after the other. The first initial of the speaker's first name is used to represent his or her turn at talk, unless there are multiple participants with the same first initial. In that case the first two initials are used. If the speaker is an instructor, the form "T/L" is used, where the second letter is the first initial of the instructor's first name. [Overlapping talk is marked with [matching square brackets across speaking turns. EMPHATIC speech is shown in uppercase, latched speech is marked with an equal sign at the end of the previous turn=, stre::tched enunciation is shown with repeated colons, pauses in talk are indicated with the number of seconds in parentheses (#s), a question mark is used to indicate raised intonation at the end of a phrase, a dash is used to indicate interrupted talk, (unclear talk) is shown in parentheses, and ((*action description*)) is shown in italics within double parentheses.
5. Direct quotes are referenced as transcript (TR) by date: [TR MM/DD/YYYY]. Observation notes are referenced as (ON MM/DD/YYYY). For simplicity, if transcripts or class notes from the same date are quoted multiple times in sequence, the date is not repeated (the reader can assume the same session is referenced as preceding quotes).

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