Modeling the Greek Aulos

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Chapter 1

Preface

This paper aims to present a model of the ancient Greek Aulos, a single or double reed woodwind instrument with a cylindrical bore, based on waveguide synthesis. All the data concerning the instruments physical properties was extracted from the archaeological remains found in the Athenian Agora excavation, in Athens, Greece.

The WG model was built in MATLAB and it was based on the scripts created by Gary P. Scavone for modeling the clarinet [14]. The changes in the code concerned mainly the entering of the Aulos physical properties (length, radius of the bore and holes, placement of holes on the bore etc.), and, in the case of the double reed Aulos, the transformation of the code from the single to the double reed.
The building of the scales was based on Letters theory of the relationship between the length of the air column, in wind instruments, and the resulting interval, and on Schlesingers theory of the moving mouthpiece according to which the performer could play different tetrachords using just 5 finger-holes by adjusting the placement of the reed in the mouthpiece.

Evaluation of the resulting data is also offered. The paper focuses on the interpretation of the resulting sounds, based mainly on their spectrum. Furthermore, the validity of the resulted scales is also discussed to some extent. Both of the instruments are examined separately and in comparison with one another.

This study aims, for the first time, to collect and combine different data, approaches, and scientific methods that belong to three different fields (archaeology, musicology and music technology), in order to create a model of an extinct instrument. By doing so, it offers a new approach on the testing of the validity of ancient Greek music theory. Archaeologists and musicologists can now base their studies in more accurate models of the different instruments, and check how and whether the theory can be put to practice.
Chapter 2

Introduction

2.1 The ancient Greek music Heritage

Music in ancient Greece[8]

The word music had a much wider meaning to the ancient Greeks than it has to us nowadays. It derives from the word Muse, which, in the classical mythology, was a word to characterize any of the 9 goddesses who protected the arts and the science. Due to its divine origins, music was considered to have magic powers, such as healing illnesses and purifying the body and the mind.

The word was used to describe not only the melody of a tune, but also its relationship to lyrics and movement. The melody and rhythm of a song
were strongly correlated with the intervals and rhythm of the words in the poem it embellished. In all the events associated with music, such as religious ceremonies, drama and music contests, the presence of singers narrating, while accompanying their stories with music and in some cases movement, was a common sight.

The first one known to have associated music with other sciences was Pythagoras and his students who declared that music and arithmetic were inseparable, and that there had to be a straightforward relation between the laws that ruled the pitch and rhythm, and those of the order of the universe. Several centuries later, Claudius Ptolemy, a leading astronomer, but also a music theorist, still supported the Pythagorean theory, and developed a mathematical system that could describe both music intervals and the behavior of the planets in our system.

Platos and Aristotles work and teaching, on the other hand, were based on the conviction that music had the ability to affect personality. Both agreed that the ideal person should have been the result of a public educational system that was based on gymnastics and music. Only certain types of music were considered ideal, though. Melodies that expressed softness and indolence, or ones with multiple tones, complex scales and rhythms, and blended tetrachords were excluded from Platos Republic along with their supporters.
In general, music in ancient Greece could be divided into two categories: the one that created a feeling of calmness and spiritual rising and was associated with the worship of Apollo, and the one that created enthusiasm and passion, and was associated with Dionysus. Apollo’s characteristic instrument was the Lyre, or the Kithara (a larger lyre), which had 5 to 7 strings, and his poetic forms were the ode and the epic, while Dionysus’ was the Aulos, a single or double reed, woodwind instrument with two bores and a piercing sound, which was used to accompany the singing of dithyrambs and dramas.

Evidence from at least the 6th century B.C. support the theory that both the Lyre and the Aulos were played at solo performances, and that ensembles consisting of both of them didn’t exist, because they were not considered a good match. The music performed was monophonic, with some elements of heterophony, played either in unison or an octave apart from the sung line. Yet, neither heterophony, nor singing in octaves can be considered evidence of the use of polyphony, which was considered corruption, and therefore banned from the everyday practice.

The theory and scales of the ancient Greek music

The primary building block of the scales used in ancient Greek music is the tetrachord, which consisted of four notes spanning a perfect fourth [8]. Aris-
toxenus, a great music theorist of the 4th century BC, identified three different types (*genera*) of tetrachords: the *diatonic*, the *chromatic*, and the *enharmonic*. The different colors of the three genera were created by the intervals formed by the two middle notes, which were "movable", in relation to the outer two, which were "immovable" [6]. In the diatonic tetrachord the lowest interval was a semitone, and the other two were whole tones. In the chromatic one the two lower intervals were semitones and the higher a semiditone (a whole and a half tone), whereas in the enharmonic the lower two intervals were smaller than semitones, and the top one a ditone (two whole tones).

![Figure 2.1: The three tetrachord types](image)

Aristoxenus, avoided specifying the size of these intervals in a numerical manner, such as ratios, the way Pythagoras and his students were doing. His basic concept was to describe the music system used in his era based on the abilities and limitations of the human hearing, allowing therefore in his method intervals denied by the Pythagoreans, because they contradicted their mathematical concept. However, in order to describe intervals smaller than the whole tone, he divided it into 12 equal parts, which he used as a measurement
Tetrachords could be combined to create one or two octave scales in two different ways:

1. If the last note of the one tetrachord was the same as the first of the next tetrachord, they were considered *conjunct* whereas,

2. If there was a whole tone separating them, the tetrachords were considered *disjunct*.

Eventually, a structure called *The Greater Perfect System*, was developed, which consisted of four disjunct and conjunct tetrachords and an extra, added tone called *proslamvanomenos*. This was a two-octave system.

![Diagram of The Greater Perfect System](image)

**Figure 2.2: The Greater Perfect System**

It is believed that the ethnic names known today (*Dorian, Phrygian, Lydian* etc.) were not applied to the octave scales by Aristoxenus and his students, but by other theorists who were trying to describe their different colors
in cultural terms. Yet, Aristoxenians have shown that those scales can simply be regarded as segments of the natural form of The Greater Perfect System.
2.2 Reconstruction of an Aulos

Both pictures on the surfaces of discovered ancient vases, and the archaeological remains themselves indicate that the Aulos was a woodwind instrument with two cylindrical bores and a single or double reed. The body of the instrument was made out of reed, just like the mouthpiece, and the whole bore was just one piece. On the Classical and the Hellenistic times bone and ivory was also used as a material of the bore for some instruments, and when keywork was developed, manufacturers covered the wooden bore with a layer of bronze or silver. These observations are also supported by the literature of those eras [11].

Yet, although so much information is provided about the Aulos, compared to the rest of the ancient Greek instruments, there are many difficulties in interpreting these evidences when it comes to reconstructing an instrument and studying its properties, which result from the kind of fragments we have available nowadays [12]. First of all, although literature suggests that the body of the instrument was of reed -a very perishable material-, all the discovered remains are of bone or ivory. Since the size of the appropriate bones was fixed and considerably smaller than the whole body of an Aulos, its bore was split into smaller sections, which fit into one another. The reconstruction of a multipart instrument from fragments is far more difficult to accomplish,
because it is almost impossible to find in an excavation site all the parts that belonged to the same bore.

Things get even more complicated because the Aulos was a double bore instrument. In other words, it had two pipes that sounded together. This means that in order to collect information about the types of sounds produced, its range and its playing techniques, we not only need to reconstruct two pipes, but also to match them into pairs.

Finally, Landels [12] indicates another problem in the attempts to recon-
struct an Aulos, which results from the place in which the fragments were found in each excavation. He claims that one cannot easily isolate that piece of information from the rest of the data that is taken into account when studying a part of an object. For example, in the case of the fragments from the Athenian Agora, or even Delos and Corinth, the great majority of them were found among rubbish deposits. This, according to Landels, questions the appropriateness of the fragments to be used as models for the reconstruction of a full functioning instruments, since chances are that they were already damaged and dysfunctional and therefore disposed.

For the creation of an acoustic model of the instrument in this paper, the necessary physical properties of its parts were extracted from fragments found in the Athenian Agora excavation in Athens, Greece. The instrument, here, will be divided into four parts that correspond to the types of fragments found in that site: 1) the mouthpiece (reed and bulb), 2) the upper part of the bore, 3) the middle part of the bore, and 4) the low-end part of the bore.

1. **The mouthpiece:**

   There are no remains of the reeds used in the Aulos available. All assumptions about the type of mouthpiece are based on the pictures of Aulos on vases that were found around Greece, which according to Landels [11] indicate the use of a double reed attached on one or two bulbs.
Yet, the possibility of a single reed is still under consideration in other studies [18].

The bulb, which takes its name from its shape, is considered to be the end part of the mouthpiece. Usually one or two of them were used. Its length is estimated to be around 9 cm and it had a socket at one end and a spigot at the other to fit to the other parts. It has been observed that, at least for the instruments that were made of bone or ivory, the inside diameter of the part does not vary with the outside curvature, which served for cosmetic purposes basically.

2. *The upper part of the bore*:
Of a varying length and diameter, the remains were made of bone or ivory. In the Athenian Aulos, this part contains always four of the five holes of the instrument (three finger-holes and a thumb-hole). The holes are in most of the cases totally circular with the same radius.

An interesting observation is that in the available remains, although the three finger-holes are in line with one another, the thumb-hole is almost never diametrically opposite to them. It is placed slightly to left or to the right of the bore. This according to Landels [11] can be an indication of whether this part belonged to the left or to the right bore of the Aulos.

3. *The middle part of the bore*:

![Figure 2.6: The middle part of the Aulos’ bore](image)

Of a varying length and diameter, the remains were made of bone or ivory. It has the last finger-hole of the Athenian Aulos. The fact that it has a socket in one end and a spigot at the other indicates that it was not the end part of the instrument.
One possible reason concerning why there had to be an extra part for the fifth hole is that bones of suitable size for the Aulos were available only in small lengths. Yet, Landels [11], has another approach according to which, the distance from the fourth to the fifth hole is slightly reduced if the hole that is covered by the little finger is slightly out of line with the rest. This practice, used nowadays too in instruments like flutes, is supposed to make the playing more convenient to the performer.

4. *The low-end part of the bore:*

![Figure 2.7: The low-end part of the Aulos’ bore](image)

Not much information is available about that part of the bore too. It was also made out of bone or ivory, and there are traces of a venting hole on it. Its overall radius does not vary considerably across its body to create a bell-shaped ending. Yet, later remains found indicate that at some point it was replaced by a more conical low-end part in the shape of a bell.
2.3 Physical properties of single and double-reed woodwind instruments

Acoustic properties of the single and double reed

The woodwind instrument reed generally acts as a pressure control valve at the input of the instrument (exciter). Its task is to convert the steady airflow of the players mouth cavity into a series of smaller airflows (puffs) that occur at frequencies controlled by the bore properties. This is achieved by periodically opening and closing a small slit in the mouthpiece, that exist either between the single-reed and the mouthpiece walls, or in the case of the double-reed, between the two reeds [4]. This difference in the geometry between the two types of exciters accounts in some extend for the differences in timbre of the individual woodwinds.

Figure 2.8: Single and Double Reed
It has been observed that reeds function properly in conditions where the airflow inside the bore vibrates in such a way that creates maximum pressure variation at the top end of the bore, next to the mouthpiece. Benade described this condition in his work *The Physics of Wood Winds* [5] in the following way: "In short, the operation of a cane reed calls for those vibrations in the air column that produce maximum variation of pressure at the reed end of the bore, and essentially zero fluctuations at the lower end...”

Moreover, Backus, who in 1961 first measured the vibrations of a single reed, by recreating artificially the blowing conditions, discovered that for soft notes, the slit of the reed almost never closed. On the other hand, the reed operated in such a way that for greater applied pressures, and therefore louder sounds, the gap opened and closed completely once every cycle [14].

Since the reed is controlled by pressure, that means that any changes in it, apart from affecting the aperture in the way we described above, affect the resulting tone quality/frequency too. For example, performers create the well-known *Vibrato* effect by applying periodic changes in the lip-pressure. Furthermore, changes in the frequency of the produced note, of the magnitude of a semitone or less, can be achieved by using increased or decreased lip pressure on the reed.

Finally, another characteristic of the woodwind reeds that is worth men-
tioning is the dependence of their response on the elevation of the location that
the performance is taking place. More specifically, since the reeds functionality as an air valve depends on pressure, it is reasonable according to Scavone [14] to expect changes in its response at considerably different elevations. The reed is said to become more inflexible, by the performers. This defect can be compensated by applying more oral cavity pressure than it was normally needed.

**Acoustic properties of the cylindrical bore**

The resonator of woodwind instruments, that is the bore, plays a significant role in the timbre of each one of them for several factors. First of all the produced frequency is controlled by its length and size. Moreover, the produced overtones are totally controlled by its geometry. As we’ll explain later, conical bores have all harmonics, whereas cylindrical ones have just the odd ones. Finally, the small mass and stiffness of the reed, give complete control over the vibration and the produced pitch to the bore [5].

In order for a type of bore to be musically useful, it is necessary that the ratios between the natural-mode frequencies remain constant, when the bore is cut in smaller lengths from the low open end, as by opening the tone-holes. In that way, the timbre of the instrument remains somehow the same for all
the instruments that belong to the same family. This theory is related to the ability of using the same holes for producing notes in all the instrument registers. Finally, it is also associated with the ability of over-blowing, on which it is based the performance of wind instruments.

The only type of horns that fulfill these requirements are the Bessel ones, for which the cross section of the bore is increased by some positive power of the distance from the end of the bore. This exponent distinguishes between the different shapes of horns. Benade [5] reports that, although there seems to be an infinite number of bores that satisfy that condition, one for every positive power, in practice certain limitations posed by the nature of our hearing system, reduce our choices to two, for which the exponent is equal to either zero or two. In the first case belong the cylindrical bore instruments, such as the clarinet and the flute, while in the later the conical bore ones, such as the oboe, the bassoon and the saxophone.

In practice, although all woodwind instruments have shapes that correspond to the two cases described above, none of them has a perfectly conical or cylindrical figure. Yet, accurate representation of the wave propagation in imperfectly shaped bores can be achieved by modeling these instruments in terms of cylindrical and conical sections [14].

As we mentioned previously, the type of pipe in an instrument controls the
harmonics present in its produced sound and its fundamental wavelength. It has been proved that in the case of open-open pipes the resonance frequencies of the bore can be calculated by the following formula,

\[ f_{o-o} = \frac{nc}{2L} \quad n \in N \]

where \( n \) is the number of the harmonic, \( c \) is the speed of air and \( L \) is the length of the bore, whereas in the case of open-closed ones by:

\[ f_{o-c} = \frac{(2n - 1)c}{4L} \quad n \in N \]

From the above equation we extract the information that open-open pipes have a fundamental wavelength equal to two times the bores length and partials at integer multiples of the fundamental frequency, while open-closed ones have a fundamental wavelength equal to 4 times their bore length and partials at odd integer multiples of the fundamental frequency.

In the case of finite length bores, that is for all musical instruments, traveling waves face discontinuities at the ends of the bore. This impedance mismatch causes part of the wave component to be transmitted into the discontinuous medium and part of it to be reflected back into the pipe. Thus, the wave variables of a finite length bore can be regarded as two superimposed waves, an ingoing and outgoing one.

This observation is true for both types of bores. Yet, there is an important difference in the case of the conical bores, since they are not symmetric about
their mid-point. Ingoing and outgoing wave propagation are different in this case. The impedance of the waves traveling towards the end of the bore is the complex conjugate of that of waves traveling away from the bore-end [14].

Finally, we have to mention that the waves inside a pipe are reflected due to the described discontinuities in a frequency dependent manner. The frequency dependent coefficient $r$ is used to describe this fact and to indicate the ratio of occurrence to reflected complex amplitudes in every frequency [14].

**Acoustic properties of the tone-holes**

The amount of side-holes present on the bore of different woodwind instruments control the number of individual pitches, or scales produced by them. The simplest approximation of the role of tone-holes in sound production is to assume that a closed hole has no effect on the wave propagation, while an open one causes the bore to behave as if it was truncated at that particular place. In other words, finger-holes offer a set of alternative bore lengths to an instrument [5].

In all the music cultures that are based on the western music system of equal temperament, the tonehole placement and radii are related to the ratio of a semitone $2^{\frac{1}{12}}$. More specifically, the distance between two successive toneholes will roughly be equal to $x = 2^{\frac{1}{12}}L$, where $L$ is the space between the
hole and the upper end of the air column [14]. Generally, the placement of the holes has to be such in western instruments that the note sounding when all the holes are open is the same with the first partial of the one sounding with all the holes closed. Needless to say that in non-western music systems such relationships between the distances of the holes are not necessarily the same.

Benade [5], was among the first who tried to define the relationship between the size of the holes and their spacing. His theory was that the ratio of the air present in a closed finger-hole to that present at the part of the bore between two successive holes had to be the same throughout the instruments body. That theory was confirmed by the structure of western, classical, woodwind instruments, whose holes get bigger in radius as we move further down on the instruments bores.

The holes dont just serve as cutting off mechanism of the bores in convenient distances from excitation, but they also play an important role on the quality of the produced sounds. It has been discovered that closed holes act, actually, as very sharp, low-pass filters that discriminate against the high components of the vibration frequencies created by the reed. The propagation wavenumber below that cut off frequency is real, whereas the one above it is imaginary. Benade [5] made also the observation that only the first couple of open holes can affect the behavior of the low frequency components of the
woodwinds.

According to Scavone [14], altering the size of the finger-holes can influence the resulting tones in different ways:

1. Increasing the radius of a hole will cause all the resonant frequency components below the cutoff frequency to rise.

2. Increasing the tonehole height will cause all the resonant frequency components below the cutoff frequency to lower.

3. Reducing the keypad height of the first couple of holes on an instrument's bore will lower the frequency on its first harmonic.

These alterations, along with their affect of the resulting tone, are known to and exercised by the woodwind manufacturers for a very long time. Yet, they are not the only types of changes on the quality of the sound available. The performer has also great control over the quality of the produced note. Attack variations, for example, are controlled by the applied mouth pressure. Slight changes on the pitch of the note are also feasible by leaving parts of the otherwise closed holes halfway open. Finally, even the production of multiphonics can be made possible, with the use of non-traditional fingering.
Chapter 3

Digital waveguide model of the Aulos

3.1 Methodology

Extraction of Physical Measurements

Of all the archaeological remains of ancient Greek musical instruments available nowadays, those of the Auloi are the most informative. Yet the reconstruction of a complete instrument out of those pieces remains a very difficult, if not impossible, task, for reasons explained earlier. For the creation of an acoustic model of the instrument in this paper, the necessary physical properties of its parts were extracted from fragments found in the Athenian Agora.
excavation in Athens, Greece. [11] More specifically, fragments A (Inv. BI 593), C (Inv. BI 579), I (Inv. BI 630) and H (Inv. BI 594) were used as models of the Aulos bulbs, the upper, middle and low-end part of the bore respectively.

Two different models were created, one with a single and one with a double reed, both of which consist of a mouthpiece with a reed and two bulbs, and the three-part bore. The physical measurements of the fragments length, radius, number, placement and size of the holes were extracted from Landels detailed study and scaled graphs available at [11].

An estimation of the reeds length was based on Letters purely mathematical approach of calculating the length on missing parts of an instrument based on the relationship between the frequencies of two notes and the resulting interval. [13] An interval can be expressed as the ratio of the higher frequency to the lower one, which is equal to the length of the air column of the lower note to that of the higher one. If one assumes that two holes in an instrument are a certain interval apart, then the length of the reed can be easily extracted.

\[
\frac{x + L_{low}}{x + L_{high}} = \text{Interval} \tag{3.1}
\]

Where \(x\) is the length of the reed and \(L_{low}\) and \(L_{high}\) the distance from the top of the bulb to the lower and higher hole respectively and \(\text{Interval}\) is the

\[\text{A description of the data and the calculations done is available in Appendix A.}\]
ratio of the resulting interval

For example if we assume that the interval between the first and the fourth hole of the modeled Aulos is a perfect fourth, then:

\[
\frac{x + L_1}{x + L_4} = \frac{4}{3} \iff x = 0.037m
\]

Following that theory, five different reed lengths were calculated based on the assumptions that the distances between different holes were a perfect fourth or a perfect fifth. By combbing that data to the already known distances between the holes on the instruments bore, five different music scales were calculated. The intervals between the produced frequencies of these scales were converted into cents and compared to the known from the literature ancient Greek scales\(^2\).

One can easily see that Letters approach is oversimplified and cannot be regarded as an accurate representation of the scales produced by an instrument, since it completely ignores factors that can considerably affect the pitch of a note, like the radius of the instruments bore, the size of the holes, the affect of the reed itself on the resulting tone, and the control each player has over the produced sound. Nevertheless, it gives an estimation of at least the length of the reed and the resulting rough length of the instrument, which were used for the waveguide digital model of the Aulos.

\(^2\)All the calculations are available in Chapter 3
Digital waveguide modeling

The usual method of examining and modeling the acoustic behavior of wind and string instruments is by dividing them into two basic parts that interact to create the sound, the resonator and the exciter. In the case of the Aulos, these will be the air column inside the bore and the reed respectively. In practice, such separation is impossible since there are no clear boundaries between these systems. Yet, this simple model has been used in many studies, [1], [3], [9], [14], with considerably accurate results.

The Digital Waveguide (DW) technique simulates the wave propagation along the instrument, in case of a woodwind one, along its bore, using delay-lines. In this project the DW model and the related Matlab script were based on the work done by Gary P. Scavone titled *An Acoustic Analysis of Single-Reed Woodwind Instruments with an Emphasis on Design and Performance Issues and Digital Waveguide Modeling Techniques* [14], and therefore the structure of the model is the following: a non-linear excitation function models the behavior of the reed, a bi-directional delay line models the resonator and a digital reflection filter models the discontinuities created at the open end of the pipe.

The script was modified to allow modeling of a 5 finger-hole plus a venting hole instrument. The radii of the bore and holes were altered to match the
ones of the Aulos and in the case of the double-reed instrument different constants and equations were added to describe the behavior of the new excitation mechanism. Also, the two-port waveguide implementation, which utilizes four second-order filter operators per hole, was picked to model the tone-holes.

Finally, the ability to choose between 5 different tunings of the instrument and to record samples of a specified duration of the created sound was also included in the scripts along with some error checking messages and default values.

A more detailed description of the DW models of each of the instrument’s parts follows.


3.2 Modeling the mouthpiece of a single and of a double reed Aulos

The reed and the mouthpiece in woodwind instruments act as a pressured control valve of the volume flow entering the reed, that controls and maintains the oscillations of the air column in the bore at frequencies corresponding to the resonant ones of the pipe. The opening area of the reed is controlled by the difference in pressure $P_\Delta$ between the inside of the reed area and the mouth cavity and can be approximated with the following model

$$P_\Delta = P_{oc} - P_r = k(S_0 - S) \quad (3.2)$$

where $P_{oc}$ and $P_r$ are the pressure of the oral cavity and of the reed respectively, $k$ is the stiffness constant of the spring, and $S_0$ is the opening area of the reed at rest. For simplicity in these cases the oral cavity is regarded as large reservoir of constant pressure and zero volume flow.

If we assume that the volume flow remains constant over the whole opening area of the reed, we can apply the Bernoulli theorem between the mouth and the reed to determine its flow $q_r$.

$$q_r = S \sqrt{\frac{2P_\Delta}{\rho}} \quad (3.3)$$

where $S$ is the opening area of the reed and $\rho$ the density of air.
These two mathematic formulas can apply to both single and double reed instruments since studies have shown that the displacements of the reeds in the latter case are symmetrical [9]. Yet, although the main principles are the same, as the geometry alters between the two, more profound changes have to be applied in order to transform the single reed into a double.

The single reed model

In the case of single reed instruments, like the clarinet, studies [14] [19] assume continuity between the reed and the bore pressures $P_r = P_b$. The pressure of the bore is regarded as the summation of the outgoing pressure traveling-wave component $p^+$ and the incoming pressure traveling-wave component from the bore $p^-$.

$$P_b = p^+ + p^- \quad (3.4)$$

In this case, the driving force of the reed operation is controlled by:

$$P_\Delta = P_{oc} (p^+ + p^-) \quad (3.5)$$

The reed to bore boundary is modeled by a reflection coefficient $r$, first suggested by Smith in 1986 [19], whose response varies according to the difference in pressure between the oral cavity and the bore $P_\Delta$. The reeds volume flow
can be represented as the act of $P_\Delta$ over the its acoustic impedance $Z_r$,

$$q_r = \frac{P_\Delta}{Z_r} \quad (3.6)$$

whereas the bores one as,

$$q_b = p^+ - \frac{p^-}{Z_b}, \quad (3.7)$$

where $Z_b$ is the impedance of the bore.

By assuming continuity between the reed and the bore flow $q_r = q_b$ and combing the resulting equation with (3.5), we get:

$$\frac{P_{oc} - p^+ + p^-}{Z_r} = \frac{p^+ - p^-}{Z_b} \quad (3.8)$$

which, when solved for the outgoing pressure wave $p^+$ gives:

$$p^+ = r \left( p^- - \frac{P_{oc}}{2} \right) + \frac{P_{oc}}{2} \quad (3.9)$$

where the pressure dependent coefficient $r$ is equal to $\frac{Z_r - Z_b}{Z_r + Z_b} [14]$.

Smith [19] also defined a term $P^+_\Delta$ such that

$$P^+_\Delta = \frac{P_{oc}}{2} - p^- \quad (3.10)$$

Which transformed equation (3.9) of the outgoing pressure wave into

$$p^+ = -r \left( \frac{P_{oc}}{2} - p^- \right) + \frac{P_{oc}}{2} \quad (3.11)$$

$$p^+ = -rP^+_\Delta + \frac{P_{oc}}{2} \quad (3.12)$$
In the case of woodwinds, Scavone suggested that there is a simple mathematical expression that can describe the behavior of the pressure dependent coefficient and results in a pretty realistic sound simulation of the instruments [14].

\[
    r = \begin{cases} 
        1 + m \left( P^+_\Delta - p_c \right) & \text{if } P^+_\Delta - p_c < 0 \\
        1 & \text{if } P^+_\Delta - p_c \geq 0
    \end{cases}
\]  

(3.13)

where \( p_c \) is the necessary pressure to close the reed, which, when normalized, equals to 1.

Values of \( P^+_\Delta \) greater than \( p_c \) indicate beating of the reed on the inner surface of the mouthpiece and reflection of the ingoing pressure wave \( p^- \). When the difference of \( p_c \) from \( P^+_\Delta \) is less than 1, partial reflection of \( p^- \) and partial transmission of \( P_{oc} \) into the bore occur. Finally, for values of \( P^+_\Delta \) less than 0, the result is negative flow through the reed, since the bore pressure is greater than the oral cavity one \( P_b > P_{oc} \) [14].

The double-reed model

The basic difference between single and double-reed instruments, is that, in the case of the latter, one cannot assume continuity between the pressure at the beginning of the reed and at the beginning of the bore \( P_r \neq P_b \). This is mainly due to the difference in the geometry of the double reed, which causes alterations in the volume flow. According to Almeida [1] these perturbations
are either created by jet reattachment, by formation of turbulent flow, or by limitations posed by the duck.

A basic formula that can describe the relationship between the pressure at the beginning of the reed \( P_r \) and that at the beginning of the bore \( P_b \) is given below

\[
P_r = P_b + \frac{1}{2} \rho \Psi \frac{q^2}{S_r^2}
\]  

(3.14)

where \( \rho \) is the density of the air, \( \Psi \) is an empirical constant that controls the dissipation factor, and \( S_r \) is the effective surface of the reed.

If we limit our approach to a quasi-static model, we can apply, once again, the Bernoulli formula (2), to describe the volume flow across the reed. In this case the opening of the reed \( S \) is controlled by

\[
S = 2\gamma z \ell \alpha
\]  

(3.15)

where \( \alpha \) is the ratio of the flow to the duck section, \( 2z \) is the distance between the two reeds, \( \gamma \) controls the geometry of the reed, and \( \ell \) is the reeds slit length.

The driving force \( P_\Delta \) of the double reed operation can be calculated by combining equations (1), (3), and (12) as follows:

\[
P_\Delta = P_{oc} - \left( p^+ + p^- + \frac{1}{2} \rho \Psi \frac{q^2}{S_r^2} \right)
\]  

(3.16)

By taking into account formulas (6) and (13), and the Bernoulli Theorem (2), we can eliminate the outgoing wave pressure \( p^+ \) from the previous equation,
resulting in the following relationship between $P_\Delta$, the mouth cavity pressure $P_{oc}$ and the bore impedance $Z_b$:

$$P_\Delta - P_{oc} + Z_b \left(2 \gamma z \ell \alpha \sqrt{P_\Delta \rho} \right) + 2p^- + \frac{1}{2} \Psi \frac{(2 \gamma z \ell \alpha)^2 2P_\Delta}{S_r^2} = 0 \quad (3.17)$$

If we set, for our convenience, $x = \sqrt{P_\Delta}$

$$\left(1 + \frac{4 \Psi (\gamma z \ell \alpha)^2}{S_r^2} \right) x^2 + \left( Z_b (2 \gamma z \ell \alpha) \sqrt{\frac{2}{\rho}} \right) x + (2p^- - P_{oc}) = 0 \quad (3.18)$$

and calculate the roots of the resulting second order differential equation, we will end up with two values, only one of which is of use to us, since pressure is a force and must have a positive value.

$$x = \frac{- \left( Z_b (\gamma z \ell \alpha) \sqrt{\frac{2}{\rho}} \right) + \sqrt{\left( Z_b (\gamma z \ell \alpha) \sqrt{\frac{2}{\rho}} \right)^2 - \left(1 + \frac{4 \Psi (\gamma z \ell \alpha)^2}{S_r^2} \right)(2p^- - P_{oc})}}{1 + \frac{4 \Psi (\gamma z \ell \alpha)^2}{S_r^2}} \quad (3.19)$$

The above gives us the relationship between the pressure of the oral cavity, the ingoing pressure wave $p^-$ and the reeds driving force for double reed instruments, as opposed to (9), which gives that of the single reed.

In order, now, to estimate the value of $p^+$ as a function of $P_\Delta$ and the reflection coefficient $r$, we need, once again, to assume continuity between the reed - bore volume flow.

$$(2 \gamma z \ell \alpha) \sqrt{\frac{2P_\Delta}{\rho}} = \frac{p^+ - p^-}{Z_b} \Leftrightarrow$$

$$p^+ = p^- + (2 \gamma z \ell \alpha) Z_b \sqrt{\frac{2P_\Delta}{\rho}} \quad (3.20)$$
The linear acoustic behavior of the bore can also be described in terms of the incoming and the outcoming pressure waves, which according to Almeida [1] can be easily transformed using the reflection function.

\[ p^- = rp^+ \] (3.21)

Having that in mind, the equation 18 can take its final form as follows:

\[ p^+ = \frac{2Z_b \gamma z \ell \alpha}{1 - r} \sqrt{\frac{2}{\rho}} P_{\Delta} \] (3.22)
3.3 Modeling the bore and the toneholes of the Aulos

The models of the cylindrical bore and the toneholes in this paper were taken from the PhD thesis of Gary P. Scavone, entitled *An Acoustic Analysis of Single-Reed Woodwind Instruments with an Emphasis on Design and Performance Issues and Digital Waveguide Modeling Techniques*, [14] and therefore, very little remains to be said in this section.

The waveguide synthesis of the holes is based on a model described by Keefe in 1990 that accurately presents a tonehole without taking into account any adjacent-hole interactions [14]. Each finger-hole is represented by a two-port junction function which models both series and shunt impedances. This method requires four second-order filter operations for each hole.

Such mathematical descriptions of the tonehole behavior can only explain two different hole-states, open or closed. Two approaches can be followed, in order to take into account the different performances of the holes in states between those two extremes. According to the first, the resulting filter coefficients can vary in time to compensate for the graduate opening and closing of the holes. In the case of the second, it is possible to model that action with a graduate cross-fade technique between the opening and closing filters.
Although such techniques are described in Scavones work [14] [15], none of them is actually implemented into his waveguide model of the clarinet [14], which was used in this project, leaving the program user with two choices, that of an open and of a closed hole.

As far as the Aulos bore is concerned, a very simple model of a cylinder can be achieved by a Gaussian function, whose peak is centered at around $2cL_b$, where $L_b$ is the length of the bore and $c$ is the speed of air. The width of the peak, which equals the time of the acoustic wave to travel from the reed to the end of the bore and back, is according to Almeida [1], an indication of the visco-thermal losses on the bore.

The cylindrical bore can be represented by a one-dimensional waveguide model of its reflection coefficient, in cases of linear acoustic propagation. The impedance mismatch that the traveling waves encounter at the end of the finite length bore causes part of the wave to be transmitted to the new medium and part of it to be reflected back. Therefore the bore of the Aulos is represented as two superimposed pressure waves, an incoming $p^-$ and an outgoing one $p^+$.

The output of the instrument $y$ is the summation of those two pressure waves $p^-$ and $p^-$

$$y = p^- + p^+ \quad (3.23)$$
In the case of the single reed Aulos, equation (3.23) takes the form of

\[ y_{\text{single}} = p^- + \frac{P_{oc}}{2} - P_\Delta \]  \hspace{1cm} (3.24)

which, by taking into account the fact that \( p^+ = \frac{p^-}{r} \), as explained before, and making the necessary substitutions, can take the form of:

\[ y_{\text{single}} = p^- \left( \frac{1}{r} + r \right) \]  \hspace{1cm} (3.25)

When a double reed is used, on the other hand, things are a little bit different and the final output of the instrument (3.23) takes the form of

\[ y_{\text{double}} = p^- + \frac{(2\gamma z \ell \alpha) Z_b \sqrt{2P_\Delta \rho}}{1 - r} \]  \hspace{1cm} (3.26)

where \( Z_b \) is the characteristic impedance of the bore, \( \alpha \) is the ratio of the flow to the duck section, \( 2z \) is the distance between the two reeds, \( \gamma \) controls the geometry of the reed, \( \ell \) is the reeds slit length, \( P_\Delta \) is the reeds driving force, and \( r \) is the reflectance coefficient.

Just a quick comparison of the two output-signals \( y_{\text{single}} \) and \( y_{\text{double}} \) reveals the complexity of the double-reed mechanism over that of the single-reed one. In the later case the output is just dependent on \( r \) and \( p^- \), whereas in the first one, \( y_{\text{double}} \) is a function of \( r, Z_b, p^- \) and \( P_\Delta \).
3.4 Interpretation and evaluation of the resulting data

The spectrum of the single reed Aulos, as it can be seen in figure 3.1, has a similar behavior to that of the classical clarinet. In other words, for the first six to seven partials, which are the most prominent ones based on their intensity, the odd harmonics dominate. Above 2000 Hz, though, this relationship is not that strict any more.

![Figure 3.1: The Single reed spectrum](image)

The harmonics don’t necessarily fall at integer multiples of the fundamental frequency. Instead, as one can notice there is a small degree of inharmonicity present. Furthermore, it is worth mentioning that from approximately 1800 Hz and on, every 7th to 8th harmonic is considerably louder than its neighbours. These peaks, are believed to be the resonant frequencies of the bore.

In the case of the double reed Aulos the spectrum of the sound is once
again dominated by the odd harmonics. This is perfectly understood, since it’s the shape of the bore that is believed to control the harmonic content of a sound and not the type of mouthpiece.

![Signal Spectrum](image)

**Figure 3.2: The Double reed spectrum**

Yet, as we can see in figure 3.2 this relationship is not that strict above 2000 Hz up until approximately 10 kHz. In this area we notice that the even harmonics gain intensity, and although still softer than the odd ones, do play a significant role in the high frequency content of the sound. Finally, one can notice certain patterns, which reveal the bore’s resonance frequencies, and a small degree of inharmonicity between the partials.

If we attempt, now, to compare the two spectrums (see figure 3.3), we will see the affect of the two different reeds on the sound of cylindrical bore woodwinds.

Two are the most apparent differences here. One concerns the energy present in the harmonic content of the sounds. As we notice the partials don’t
Figure 3.3: Single and Double reed spectrum comparison

decay in equal ways and to the same degree. For reasons, which are beyond the scope of this thesis, that have to do with the operation of the reeds, the spectrum of the double reed carries considerably more energy.

The other one has to do with the intonation of the two sounds, which appear in the spectrum to be slightly detuned, even though they were created on the same bore and with exactly the same fingering. We will revisit this aspect in chapter 4, where we deal with the Aulos' scales.
Chapter 4

The Aulos’ scales

For the purposes of this research paper, two different models of the Ancient Greek Aulos were built using MATLAB, a single and a double-reed one, based on the fragments found in the Athenian Agora excavation in Athens, Greece. Both of the instruments consist of a mouthpiece, of two bulbs, and a three part bore. Yet, as we said before, there is no information available on the reed of the instrument.

It is understood that an estimation of the sound of the instrument and its produced scales cannot be considered exact without the presence of the reed physical properties. Letters, therefore, came up with a purely mathematical method of estimating the reeds length and the possible scales played by an Aulos based on the relationship between the frequencies of two notes and the
resulting interval [13].

Although this approach of estimating scales cannot be treated as accurate, since it disregards all the other factors that play a significant role in the production of sound, such as the radius of the instrument’s bore, and holes, the affect of the reed itself on the resulting tone, and the control each player has over the produced sound, it was used here, as we claimed before, in order to calculate just the length $x$ of the reed.

Moreover, the presence of 5 toneholes on the bore of the instruments remains a mystery, since the ancient Greek music was based on the use of tetrachords. One may use this fact as evidence that more than one tetrachords were played with the same instrument. Yet, since resulting intervals between adjacent holes are not equal, like in the case of equal temperament, this will require some changes on the instrument’s effective length.

At first, such a fact may seem impractical, because adding to or removing parts, like bulbs for example, from the instrument while performing is unfeasible. K. Schlesinger, however, suggested the theory of the moving mouthpiece [18]. According to that, the player could move from one tetrachord to the other easily by pushing of pulling the reed in and out of the mouthpiece, affecting in that way its effective length. In such a system, the two higher holes of the lower tetrachord may be used as the two lower holes of the higher one.
What follows is a description of the calculations of the scales for the resulting single and double reed Aulos.
4.1 Calculation of the resulting scales

For the purpose of estimating the length of the reed in the Aulos, five different assumptions were made:

1. The interval between all holes open and the top three holes closed is a perfect fourth:

\[
\frac{x + L_0}{x + L_3} = \frac{4}{3} \quad \Leftrightarrow \quad x = 0.042 \text{ m}
\]

2. The interval between the first and the fourth hole is a perfect fourth:

\[
\frac{x + L_1}{x + L_4} = \frac{4}{3} \quad \Leftrightarrow \quad x = 0.037 \text{ m}
\]

3. The interval between the second and the fifth hole is a perfect fourth:

\[
\frac{x + L_2}{x + L_5} = \frac{4}{3} \quad \Leftrightarrow \quad x = 0.0729 \text{ m}
\]

4. The interval between all holes open and the top four holes closed is a perfect fifth:

\[
\frac{x + L_0}{x + L_4} = \frac{3}{2} \quad \Leftrightarrow \quad x = 0.029 \text{ m}
\]

5. The interval between the first and the fifth hole is a perfect fifth:

\[
\frac{x + L_1}{x + L_5} = \frac{3}{2} \quad \Leftrightarrow \quad x = 0.0476 \text{ m}
\]

\(^1\)A description of the lengths of each fragment used, and the distances between the holes is available in Appendix.
As one can easily see, all the reed lengths, with maybe the exception of that in the third case, fall within a reasonable range. There are two possible reasons why the length of the reed is bigger in assumption number 3. Either the selected ratio of the second to the fifth hole is not correct, or the dimensions of the selected fragment, which corresponds to the part of the bore that has the fifth hole, don’t match those of the rest of the instrument parts. Both cases are expected to occur anywhere in the model, since it is well understood that a model which is based on parts that is not sure if they belong together can only be regarded as an approximation of a real instrument and not as its accurate representation.

In any case, the adjustments of the reed length that need to be made in order for the player to move from one tetrachord to the other are of the magnitude of a centimeter, and therefore easy to achieve. This may lead us to think that Schlesinger’s theory of the moving mouthpiece can be considered possible.

The following table contains the distances from the excitation mechanism, to each hole and to the end of the bore, for every one of the assumptions described before. If we look at the resulting data, we can see that the resulting lengths for each scale are more or less the same, with the exceptions explained before. More specifically, for example, the length of the mouthpiece

49
is approximately 21 cm, and that of the bore is around 50 cm.

<table>
<thead>
<tr>
<th></th>
<th>Assum. 1</th>
<th>Assum. 2</th>
<th>Assum. 3</th>
<th>Assum. 4</th>
<th>Assum. 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{mipiece}$ (m)</td>
<td>0.221</td>
<td>0.216</td>
<td>0.2519</td>
<td>0.208</td>
<td>0.2266</td>
</tr>
<tr>
<td>$L_1$ (m)</td>
<td>0.245</td>
<td>0.24</td>
<td>0.2759</td>
<td>0.232</td>
<td>0.2506</td>
</tr>
<tr>
<td>$L_2$ (m)</td>
<td>0.27</td>
<td>0.265</td>
<td>0.3009</td>
<td>0.257</td>
<td>0.2756</td>
</tr>
<tr>
<td>$L_3$ (m)</td>
<td>0.298</td>
<td>0.293</td>
<td>0.3289</td>
<td>0.285</td>
<td>0.3036</td>
</tr>
<tr>
<td>$L_4$ (m)</td>
<td>0.325</td>
<td>0.32</td>
<td>0.3559</td>
<td>0.312</td>
<td>0.3306</td>
</tr>
<tr>
<td>$L_5$ (m)</td>
<td>0.3503</td>
<td>0.3453</td>
<td>0.3812</td>
<td>0.3373</td>
<td>0.3559</td>
</tr>
<tr>
<td>$L_{vent}$ (m)</td>
<td>0.4582</td>
<td>0.4532</td>
<td>0.4891</td>
<td>0.4452</td>
<td>0.4638</td>
</tr>
<tr>
<td>$L_{bore}$ (m)</td>
<td>0.5045</td>
<td>0.4995</td>
<td>0.5354</td>
<td>0.4915</td>
<td>0.5101</td>
</tr>
</tbody>
</table>

As far as the calculation of the resulting scales of each of the 5 assumptions is concerned, we avoided following Letters’ approach of converting the above lengths of the air columns into cents [13], because as it was proven in a test, it is really inaccurate and creates considerable deviations from the actual sounding scales of the models. Instead the following formula was used to convert from frequency ratios to cents.

$$cents = \frac{1200}{\log 2} \log \left( \frac{f_{\text{high}}}{f_{\text{low}}} \right)$$  \hspace{1cm} (4.1)

The frequencies of the first harmonic of the notes that resulted from each of the previous assumptions were calculated in MATLAB. By doing so, the estimation of the produced notes is not simply a function of lengths, but takes, also, into account factors such as the type of the reed, the length and radius of the bore, and the radius and placement of the holes on the bore, that make it more accurate. Furthermore, two sets of frequencies and interval ratios were
calculated since it was noticed that the resulting notes of the single and the double reed model had slightly different frequencies.

Following we can see the resulting frequencies of the two types of Aulos, for each of the five assumptions mentioned before. $F_0$ stands for the frequency produced when all the holes on the bore are left open, $S_r$ stands for Single reed and $D_r$ for Double reed.

<table>
<thead>
<tr>
<th></th>
<th>Assum. 1</th>
<th>Assum. 2</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$S_r$</td>
<td>$D_r$</td>
</tr>
<tr>
<td>$f_0$ (Hz)</td>
<td>295</td>
<td>287</td>
</tr>
<tr>
<td>$f_1$ (Hz)</td>
<td>270</td>
<td>273</td>
</tr>
<tr>
<td>$f_2$ (Hz)</td>
<td>244</td>
<td>242</td>
</tr>
<tr>
<td>$f_3$ (Hz)</td>
<td>226</td>
<td>228</td>
</tr>
<tr>
<td>$f_4$ (Hz)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$f_5$ (Hz)</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Assum. 3</th>
<th>Assum. 4</th>
<th>Assum. 5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S_r$</td>
<td>$D_r$</td>
<td>$S_r$</td>
</tr>
<tr>
<td>$f_0$ (Hz)</td>
<td>-</td>
<td>-</td>
<td>314</td>
</tr>
<tr>
<td>$f_1$ (Hz)</td>
<td>-</td>
<td>-</td>
<td>285</td>
</tr>
<tr>
<td>$f_2$ (Hz)</td>
<td>223</td>
<td>221</td>
<td>257</td>
</tr>
<tr>
<td>$f_3$ (Hz)</td>
<td>207</td>
<td>209</td>
<td>237</td>
</tr>
<tr>
<td>$f_4$ (Hz)</td>
<td>191</td>
<td>191</td>
<td>216</td>
</tr>
<tr>
<td>$f_5$ (Hz)</td>
<td>153</td>
<td>153</td>
<td>-</td>
</tr>
</tbody>
</table>

Moreover a calculation of the intervals in cents was performed that would let us compare the resulting scales to the known ones from the Greek literature. For each of the previous 5 cases 3, different calculations were performed: one
for the single reed model, one for the double reed model, and another one using Letters’ mathematical approach of converting air column lengths into scales.

<table>
<thead>
<tr>
<th></th>
<th>Assum. 1</th>
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<th>Assum. 2</th>
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<tbody>
<tr>
<td></td>
<td>$S_r$</td>
<td>$D_r$</td>
<td>$L$</td>
</tr>
<tr>
<td>$f_0$</td>
<td>153.3</td>
<td>86.6</td>
<td>178.5</td>
</tr>
<tr>
<td>$f_1$</td>
<td>175.3</td>
<td>208.7</td>
<td>168.2</td>
</tr>
<tr>
<td>$f_2$</td>
<td>132.7</td>
<td>103.2</td>
<td>170.8</td>
</tr>
<tr>
<td>$f_3$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$f_4$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Assum. 3</th>
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<th>Assum. 4</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$S_r$</td>
<td>$D_r$</td>
<td>$L$</td>
</tr>
<tr>
<td>$f_0$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$f_1$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$f_2$</td>
<td>128.9</td>
<td>96.7</td>
<td>154</td>
</tr>
<tr>
<td>$f_3$</td>
<td>139.3</td>
<td>155.9</td>
<td>136.6</td>
</tr>
<tr>
<td>$f_4$</td>
<td>384</td>
<td>384</td>
<td>118.9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Assum. 5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S_r$</td>
</tr>
<tr>
<td>$f_0$</td>
<td>-</td>
</tr>
<tr>
<td>$f_1$</td>
<td>-</td>
</tr>
<tr>
<td>$f_2$</td>
<td>128.3</td>
</tr>
<tr>
<td>$f_3$</td>
<td>147</td>
</tr>
<tr>
<td>$f_4$</td>
<td>401.3</td>
</tr>
</tbody>
</table>

The computed calculations above can be treated as proof of the fact that the instrument’s physical properties affect considerably the resulting tones. This is not only understood by the deviation of the calculations made with
Letters’ model from the other two, which is the most apparent, but also from
the differences between the single and the double reed instrument frequencies
and intervals. This observed variation can also be treated as evidence of the
fact that the bore of the woodwinds, although the most important, is definitely
not the only factor that affects the resulting tones.
4.2 Evaluation of the resulting scales and comparison with the known ancient Greek ones

Unfortunately, no match with any of the known ancient Greek scales was made possible in any of the above cases. This can be the result of various reasons, all of which are understandable and were more or less expected to occur at some point in the project.

First of all, the fragments used to compose this Physical Model of the Aulos don’t necessarily belong to the same instrument, or to the same bore. Most likely, they belonged to different instruments, built by various people at different years during the 5th and the 4th century B.C.

Moreover, this model utilizes only one bore, although we know that the Aulos was a two double bore instrument. The reason of that omission is that no information is available on that second bore, mainly because there are not enough fragments found to completely reconstruct an Aulos, and therefore extract with relative certainty each bores physical properties. Yet, it is understood that the presence of a second bore in this model will considerably affect the resulting tones and their intervals.

Furthermore, the waveguide approach was inevitably based on the way
acoustic properties of woodwind instruments exist now. In other words, it was based on instruments whose level of perfection, as far as their construction is concerned, is far from that of the Aulos. For example, Schlesinger [18], who was among the first to attempt the physical reconstruction of an Aulos, wrote in her report:

"The type of double reed mouthpiece in use on the primitive Greek Aulos was obviously not the highly sophisticated oboe reed of the present day, but the simple ripe of wheat or oat stalk, plucked from the cornfield".

Such a difference in the building standards is also expected to affect the produced sounds to a great extend.

Finally, this model doesn’t take into account the control that the performer has over the sound of his/her instrument. This was merely done for two reasons. First of all, the complexity that such an approach would have added to the waveguide model goes well beyond the scope of this project. Second, there is no information available, at least to the knowledge of the author, that contains information about the performing techniques of wind instruments in Ancient Greece.

However, although all the above factors discriminate against the possible accuracy of the resulting Aulos’ sound and scales, it is believed that by trial and error, the correct combinations of fragments and reed lengths, that lead
to more accurate scales, can be made. In any case, though, these results can
probably also be taken into account when the validity of the Ancient Greek
music theory, the way it is interpreted nowadays, is under question.
Chapter 5

Conclusions/ Future work

This study attempted to reconstruct the Ancient Greek Aulos based on waveguide synthesis. The two resulting models, which were based on the fragments found in the Athenian Agora excavation, and their related studies, managed to represent the Aulos to a certain extent and in way that wasn’t attempted before, at least to the knowledge of the author.

Work still remains to be done, though, in the fields of the sound realism and recreation of the ancient Greek scales. The imperfection of the instruments manufacture, as described above, has to be taken into account in the MATLAB algorithm along with some options of performance control. Furthermore, the combination of bore’s and reed’s length, along with the appropriate fingering for the creation of the Aulos’ scales, has to be revisited.
Chapter 6

Appendix

6.1 Physical Measurements of the Athenian Aulos fragments

The following table contains data that was extracted from Landels’ description of the Aulos’ fragments found in the Athenian Agora excavation [11]. For the purposes of this paper fragments A, C, I and H were used.

<table>
<thead>
<tr>
<th></th>
<th>Mouthpiece</th>
<th>Upper Bore</th>
<th>Middle Bore</th>
<th>Low end Bore</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (cm)</td>
<td>8.95</td>
<td>11.3</td>
<td>9.15</td>
<td>7.9</td>
</tr>
<tr>
<td>Bore radius (cm)</td>
<td>1.85/2</td>
<td>1.6/2</td>
<td>1.8/2</td>
<td>1.8/2</td>
</tr>
<tr>
<td>Hole radius (cm)</td>
<td>-</td>
<td>0.85/2</td>
<td>0.95/2</td>
<td>-</td>
</tr>
</tbody>
</table>

Distance form the center of each hole to the upper part of the corresponding fragment:
Fragment C:

\[ H_1 = 0.024 \text{m}, \ H_2 = 0.049 \text{m}, \ H_3 = 0.077 \text{m}, \ IH_4 = 0.104 \text{m} \]

Fragment I:

\[ H_5 = 0.0163 \text{ m} \]

Fragment H:

\[ H_{\text{vent}} = 0.0327 \text{ m} \]
6.2 Matlab Code

Single Reed Aulos

function y = aulos_singlereed(finger, scale, dur, filename)

% This Matlab function implements a digital waveguide model of the
% single-reed Aulos.
% Inputs:
%   finger: 6 element vector that controls the tonehole fingering. a "0"
%           indicates an open hole and a "1" indicates a closed hole.
%           default value = all holes open.
%   scale: the tuning of the aulos. takes one of the values: 1, 2, 3, 4
%           and 5, which correspond to 5 different default scales.
%           The scales are computed as distances from excitation to
%           toneholes and to the end of the bore in meters.
%   dur: duration in seconds of the resulting signal. default value =
%           1 second.
%   filename: name of the resulting .wav file. default value =
%           "test.wav"
% Output:
% .wav file
% The code was based on a matlab script of a digital waveguide
% clarinet model by Gary P. Scavone, CCRMA, Stanford University,
% 12 March 1997.
% By Areti Andreopoulou, New York University, May 2008.

% **** Error Checking **** %

if (nargin < 4)
    filename = 'test';
    disp('the name of the resulting .wav file is set to test.wav!')
end;

if (ischar(filename) ~= 1)
    disp('Error: filename must be a string!')
    filename = 'test';
    disp('the filename is reset to test.wav!')
end;
end;

if (nargin < 3)
    dur = 0.5;
    disp('the duration of the tone is set to 0.5 sec!')
end;

if (ischar(dur) == 1)
    disp('Error: duration must be a number!')
    return;
end;

if (nargin < 2)
    scale = 1;
    disp('the first sample scale is selected!')
end;

if (scale<=0 || scale > 5)
    disp('Error: scale must have one of the following values: 1, 2, 3, 4 or 5!')
    return;
end;

if ( scale / floor(scale) ~= 1)
    disp('Error: scale must have one of the following values: 1, 2, 3, 4 or 5!')
    return;
end;

if (ischar(scale) == 1)
    disp('Error: scale must have one of the following values: 1, 2, 3, 4 or 5!')
    return;
end;

if (nargin < 1)
    finger = [0, 0, 0, 0, 0, 0];
    disp('finger is set to [0, 0, 0, 0, 0, 0]!')
end;
if (ischar(finger) == 1)
    disp('Error: finger must be a 6 element vector')
    return;
end;

if (length(finger) ~= 6)
    disp('Error: finger must be a 6 element vector')
    return;
end;

if (sum(finger)<0 || sum(finger)>6)
    disp('Error: the vector can be filled only with ones and zeros')
    return;
end;

% *** Signal Parameters *** %

fs = 44100;    % sampling rate
N = dur*fs;    % number of samples to compute

% ******* Air Column Parameters ******* %

c = 347.23;    % speed of sound in air (meters/second)
ra = 0.0175/2; % radius of bore (meters) %%% ARETI
tw = 0.0034;   % shortest tonehole height (meters)
rc = 0.0005;   % tonehole radius of curvature (meters)

% **** Distances From Excitation To Toneholes & End (m) **** %

if (scale == 1)
    L = [0.245, 0.27, 0.298, 0.325, 0.3503, 0.4582, 0.5045];
end;

if (scale == 2)
    L = [0.24, 0.265, 0.293, 0.32, 0.3453, 0.4532, 0.4995];
end;

if (scale == 3)
    L = [0.2759, 0.3009, 0.3289, 0.3559, 0.3812, 0.4891, 0.5354];
end;
if (scale == 4)
    L = [0.232, 0.257, 0.285, 0.312, 0.3373, 0.4452, 0.4915];
end;

if (scale == 5)
    L = [0.2506, 0.2756, 0.3036, 0.3306, 0.3559, 0.4638, 0.5101];
end;

% ******* Tonehole Radii (meters) ******* %
rb = [0.009 0.009 0.009 0.009 0.009 0.009]/2;

% **** Rounded Distances Between Elements (samples) **** %
D = round(fs*[L(1) L(2:length(L))-L(1:length(L)-1)]/c);

% ******* Tonehole Filter Coefficient Vectors ******* %
br_th = zeros(length(rb),3);
ar_th = zeros(length(rb),3);
b_t_th = zeros(length(rb),3);
at_th = zeros(length(rb),3);
zerup_th = zeros(length(rb),2);
zup_th = zeros(length(rb),2);
zrdn_th = zeros(length(rb),2);
ztn_th = zeros(length(rb),2);

% ******* Closed Tonehole 2-Port Filter Coefficients ******* %
for i=1:length(finger),
    if finger(i)==1,
        [br_th(i,:),ar_th(i,:),bt_th(i,:),at_th(i,:)] = closhole(ra,rb(i),tw,fs,0);
    else [br_th(i,:),ar_th(i,1:2),bt_th(i,:),at_th(i,:)] =
        openhole(ra,rb(i),tw,rc,fs,0);
    end;
end;

% ******* Open-end Reflectance Filter ******* %
[b_open,a_open] = openpipe(1,2,ra,fs,0);
z_open = [0 0];

% ******* Boundary-Layer Losses ******* %
% The air column lengths between toneholes are short, so that losses
% will be small in these sections. Further, such losses can be
% commuted with fractional-delay filters (which are not incorporated
% here). In this model, boundary-layer losses are only included for
% the first, long cylindrical bore section.

[b_boundary,a_boundary] = boundary(2,2,ra,fs,D(1),0);
z_boundary = zeros(2,2);

% ******* Initialize delay lines ******* %

y = zeros(1,N); % initialize output vector
d11 = zeros(2,D(1));
d12 = zeros(2,D(2));
d13 = zeros(2,D(3));
d14 = zeros(2,D(4));
d15 = zeros(2,D(5));
d16 = zeros(2,D(6));
d17 = zeros(2,D(7));
ptr = ones(length(rb)+1);
upin2pt = zeros(size(rb));
dnin2pt = zeros(size(rb));

p_oc = 0; % initial oral cavity pressure
pinc = 0.01; % oral cavity pressure increment

% **************************** %
%  % %
% Delay Line  %
% |------------------|  %
% ^ %
% pointer  %
% %
% >>--- pointer increments ---->>  %
% %
The pointer initially points to each delay line output. We can take the output and calculate a new input value which is placed where the output was taken from. The pointer is then incremented and the process repeated.

% ******* Run Loop Start ******* %

for i = 1:N,
    pbminus = dl1(2,ptr(1));
    upin2pt = [dl1(1,ptr(1)), dl2(1,ptr(2)), dl3(1, ptr(3)), dl4(1, ptr(4)),
                dl5(1, ptr(5)), dl6(1, ptr(6))];
    dnin2pt = [dl2(2,ptr(2)), dl3(2,ptr(3)), dl4(2, ptr(4)), dl5(2, ptr(5)),
                dl6(2, ptr(6)), dl7(2, ptr(7))];

    % ******* Boundary-Layer Loss Filter Calculations ******* %
    [upin2pt(1),z_boundary(1,:)] = filter(b_boundary,a_boundary,
                                        upin2pt(1),z_boundary(1,:));
    [pbminus,z_boundary(2,:)] = filter(b_boundary,a_boundary,
                                        pbminus,z_boundary(2,:));

    % ******* Open-end Filter Calculations ******* %
    [dl7(2,ptr(7)),z_open] = filter(b_open,a_open,dl7(1,ptr(7)),z_open);

    % ***** First Tonehole Calculations ***** %
    [temp1,zrup_th(1,:)] = filter(br_th(1,:),ar_th(1,:),upin2pt(1),
                                 zrup_th(1,:));
    [temp2,ztdn_th(1,:)] = filter(bt_th(1,:),at_th(1,:),dnin2pt(1),
                                 ztdn_th(1,:));
    dl1(2,ptr(1)) = temp1 + temp2;
    [temp1,zrdn_th(1,:)] = filter(br_th(1,:),ar_th(1,:),dnin2pt(1),
                                zrdn_th(1,:));
    [temp2,ztup_th(1,:)] = filter(bt_th(1,:),at_th(1,:),upin2pt(1),
                                ztup_th(1,:));
    dl2(1,ptr(2)) = temp1 + temp2;
% ***** Second Tonehole Calculations ***** %

[temp1,zrup_th(2,:)] = filter(br_th(2,:),ar_th(2,:),upin2pt(2),
  zrup_th(2,:));
[temp2,ztdn_th(2,:)] = filter(bt_th(2,:),at_th(2,:),dnin2pt(2),
  ztdn_th(2,:));
dl2(2,ptr(2)) = temp1 + temp2;
[temp1,zrdn_th(2,:)] = filter(br_th(2,:),ar_th(2,:),dnin2pt(2),
  zrdn_th(2,:));
[temp2,ztup_th(2,:)] = filter(bt_th(2,:),at_th(2,:),upin2pt(2),
  ztup_th(2,:));
dl3(1,ptr(3)) = temp1 + temp2;

% ***** Third Tonehole Calculations ***** %

[temp1,zrup_th(3,:)] = filter(br_th(3,:),ar_th(3,:),upin2pt(3),
  zrup_th(3,:));
[temp2,ztdn_th(3,:)] = filter(bt_th(3,:),at_th(3,:),dnin2pt(3),
  ztdn_th(3,:));
dl3(2,ptr(3)) = temp1 + temp2;
[temp1,zrdn_th(3,:)] = filter(br_th(3,:),ar_th(3,:),dnin2pt(3),
  zrdn_th(3,:));
[temp2,ztup_th(3,:)] = filter(bt_th(3,:),at_th(3,:),upin2pt(3),
  ztup_th(3,:));
dl4(1,ptr(4)) = temp1 + temp2;

% ***** Fourth Tonehole Calculations ***** %

[temp1,zrup_th(4,:)] = filter(br_th(4,:),ar_th(4,:),upin2pt(4),
  zrup_th(4,:));
[temp2,ztdn_th(4,:)] = filter(bt_th(4,:),at_th(4,:),dnin2pt(4),
  ztdn_th(4,:));
dl4(2,ptr(4)) = temp1 + temp2;
[temp1,zrdn_th(4,:)] = filter(br_th(4,:),ar_th(4,:),dnin2pt(4),
  zrdn_th(4,:));
[temp2,ztup_th(4,:)] = filter(bt_th(4,:),at_th(4,:),upin2pt(4),
  ztup_th(4,:));
dl5(1,ptr(5)) = temp1 + temp2;
% ***** Fifth Tonehole Calculations ***** 

[temp1,zrup_th(5,:)] = filter(br_th(5,:),ar_th(5,:),upin2pt(5),
zrup_th(5,:));
[temp2,ztdn_th(5,:)] = filter(bt_th(5,:),at_th(5,:),dnin2pt(5),
ztdn_th(5,:));
dl5(2,ptr(5)) = temp1 + temp2;
[temp1,zrdn_th(5,:)] = filter(br_th(5,:),ar_th(5,:),dnin2pt(5),
zrdn_th(5,:));
[temp2,ztup_th(5,:)] = filter(bt_th(5,:),at_th(5,:),upin2pt(5),
ztup_th(5,:));
dl6(1,ptr(6)) = temp1 + temp2;

% ***** Sixth Tonehole Calculations ***** 

[temp1,zrup_th(6,:)] = filter(br_th(6,:),ar_th(6,:),upin2pt(6),
zrup_th(6,:));
[temp2,ztdn_th(6,:)] = filter(bt_th(6,:),at_th(6,:),dnin2pt(6),
ztdn_th(6,:));
dl6(2,ptr(6)) = temp1 + temp2;
[temp1,zrdn_th(6,:)] = filter(br_th(6,:),ar_th(6,:),dnin2pt(6),
zrdn_th(6,:));
[temp2,ztup_th(6,:)] = filter(bt_th(6,:),at_th(6,:),upin2pt(6),
ztup_th(6,:));
dl7(1,ptr(7)) = temp1 + temp2;

% ******* Excitation Calculations ******* 

p_delta = 0.5*p_oc - pbminus;

% ****** Pressure-Dependent Reflection Coefficient ****** 

refl = 0.7 + 0.4*p_delta;
if refl > 1.0
    refl = 1.0;
end;

dl1(1,ptr(1)) = 0.5*p_oc - p_delta*refl;

% The system output is determined at the input to the air
% column. The oral cavity is incremented until it reaches % the desired value.

\[ y(i) = p_{bminus} + d_{11}(1, \text{ptr}(1)) \]

if \( p_{oc} < 1 \),
    \[ p_{oc} = p_{oc} + p_{inc} \]
end;

% ***** Increment Pointers & Check Limits ***** %

for \( j=1:length(L) \),
    ptr(j) = ptr(j) + 1;
    if ptr(j) > D(j),
        ptr(j) = 1;
    end;
end;

% *** Plot, Play and Wavwrite *** %

\[ \text{norm}_y = y./\text{max}(\text{abs}(y)) \];

figure(1)
plot(norm_y, 'm')
title('Signal Output');
xlabel('Samples');
ylabel('Signal Level');

sound(norm_y, fs);
wavwrite(norm_y, fs, 16, filename);

\[ \text{SIG} = \text{abs}(\text{fft}(y/\text{max}(\text{abs}(y)), 40000)) \]
\[ X = 20*\log10(\text{SIG}) \]

figure(2)
plot(X(1:(length(X)/2)+1));
title('Signal Spectrum');
xlabel('Freq');
ylabel('dB');
Double Reed Aulos

function y = aulos_doublereed(finger, scale, dur, filename)

% This Matlab function implements a digital waveguide model of the
% double-reed Aulos.
% Inputs:
%   finger: 6 element vector that controls the tonehole fingering. a "0"
%           indicates an open hole and a "1" indicates a closed hole.
%           default value = all holes open.
%   scale: the tuning of the aulos. takes one of the values: 1, 2, 3, 4
%           and 5, which correspond to 5 different default scales.
%           The scales are computed as distances from excitation to
%           toneholes and to the end of the bore in meters.
%   dur: duration in seconds of the resulting signal. default value =
%       1 second.
%   filename: name of the resulting .wav file. default value =
%       "test.wav"
% Output:
%   .wav file
% The code was based on a matlab script of a digital waveguide
% clarinet model by Gary P. Scavone, CCRMA, Stanford University,
% 12 March 1997.
% By Areti Andreopoulou, New York University, May 2008.
%
% ***** Error Checking ***** %

if (nargin < 4)
    filename = 'test';
    disp('the name of the resulting .wav file is set to test.wav!')
end;

if (ischar(filename) ~= 1)
    disp('Error: filename must be a string!')
    filename = 'test';
    disp('the filename is reset to test.wav!')
end;


if (nargin < 3)
    dur = 0.5;
    disp('the duration of the tone is set to 0.5 sec!')
end;

if (ischar(dur) == 1)
    disp('Error: duration must be a number!')
    return;
end;

if (nargin < 2)
    scale = 1;
    disp('the first sample scale is selected!')
end;

if (scale<=0 || scale > 5)
    disp('Error: scale must have one of the following values: 1, 2, 3, 4 or 5!')
    return;
end;

if ( scale / floor(scale) ~= 1)
    disp('Error: scale must have one of the following values: 1, 2, 3, 4 or 5!')
    return;
end;

if (ischar(scale) == 1)
    disp('Error: scale must have one of the following values: 1, 2, 3, 4 or 5!')
    return;
end;

if (nargin < 1)
    finger = [0, 0, 0, 0, 0, 0];
    disp('finger is set to [0, 0, 0, 0, 0, 0]!')
end;
if (ischar(finger) == 1)
    disp('Error: finger must be a 6 element vector')
    return;
end;

if (length(finger) ~= 6)
    disp('Error: finger must be a 6 element vector')
    return;
end;

if (sum(finger)<0 || sum(finger)>6)
    disp('Error: the vector can be filled only with ones and zeros')
    return;
end;

% *** Signal Parameters *** %

fs = 44100; % sampling rate
N = dur*fs; % number of samples to compute

% ******* Air Column Parameters *******%

c = 347.23; % speed of sound in air (meters/second)
ra = 0.0175/2; % radius of bore (meters)
tw = 0.0034; % shortest tonehole height (meters)
rc = 0.0005; % tonehole radius of curvature (meters)

% ****** Physical constants ****** %

rho = 1.1769; % air density
alfa = 0.6; % ratio of the flow to the duct section
l = 4.6*10^-3; % slit length of the reed
z = 0.5*10^-3; % distance between the reeds
gama = 1/2; % geometry of the reed
Sr = 2*(5*10^-3)*0.042; % area of the reed
psi = 100; % dissipation factor
Sb = pi*ra^2; % area of the bore
Zob = rho*c/Sb; % the characteristic impedance of the bore
% ******* Distances From Excitation To Toneholes & End (m) ******* %

if (scale == 1)
    L = [0.245, 0.27, 0.298, 0.325, 0.3503, 0.4582, 0.5045];
end;

if (scale == 2)
    L = [0.24, 0.265, 0.293, 0.32, 0.3453, 0.4532, 0.4995];
end;

if (scale == 3)
    L = [0.2759, 0.3009, 0.3289, 0.3559, 0.3812, 0.4891, 0.5354];
end;

if (scale == 4)
    L = [0.232, 0.257, 0.285, 0.312, 0.3373, 0.4452, 0.4915];
end;

if (scale == 5)
    L = [0.2506, 0.2756, 0.3036, 0.3306, 0.3559, 0.4638, 0.5101];
end;

% ******* Tonehole Radii (meters) ******* %

rb = [0.009 0.009 0.009 0.009 0.009 0.009]/2;

% ******* Rounded Distances Between Elements (samples) ******* %

D = round(fs*[L(1) L(2:length(L))-L(1:length(L)-1)]/c);

% *** Initialize Empty Vectors *** %

PB = [];
DL = [];

% ******* Tonehole Filter Coefficient Vectors ******* %

br_th = zeros(length(rb),3);
ar_th = zeros(length(rb),3);
btt_th = zeros(length(rb),3);
at_th = zeros(length(rb),3);
zrup_th = zeros(length(rb),2);
ztrup_th = zeros(length(rb),2);
zrdn_th = zeros(length(rb),2);
ztdn_th = zeros(length(rb),2);

% ******* Closed Tonehole 2-Port Filter Coefficients ******* %
for i=1:length(finger),
  if finger(i)==1,
    [br_th(i,:),ar_th(i,:),bt_th(i,:),at_th(i,:)] =
    closhole(ra,rb(i),tw,fs,0);
  else
    [br_th(i,:),ar_th(i,1:2),bt_th(i,:),at_th(i,:)] =
    openhole(ra,rb(i),tw,rc,fs,0);
  end;
end;

% ******* Open-end Reflectance Filter ******* %
[b_open,a_open] = openpipe(1,2,ra,fs,0);
z_open = [0 0];

% ******* Boundary-Layer Losses ******* %
% The air column lengths between toneholes are short, so that losses
% will be small in these sections. Further, such losses can be
% commuted with fractional-delay filters (which are not incorporated
% here). In this model, boundary-layer losses are only included for
% the first, long cylindrical bore section.

[b_boundary,a_boundary] = boundary(2,2,ra,fs,D(1),0);
z_boundary = zeros(2,2);

% ******* Initialize delay lines ******* %
y = zeros(1,N); % initialize output vector
dl1 = zeros(2,D(1));
dl2 = zeros(2,D(2));
dl3 = zeros(2,D(3));
dl4 = zeros(2,D(4));
dl5 = zeros(2,D(5));
dl6 = zeros(2,D(6));
dl7 = zeros(2,D(7));

ptr = ones(length(rb)+1);
upin2pt = zeros(size(rb));
dnin2pt = zeros(size(rb));

p_oc = 0; % initial oral cavity pressure
pinc = 0.01; % oral cavity pressure increment

% ****************************************************************************** %
%delay line %
%------------------------------------------ %
% ^ %
% pointer %
% %
% >>--- pointer increments ---> %
% %
%****************************************************************************** %

% The pointer initially points to each delay line output. % We can take the output and calculate a new input value % which is placed where the output was taken from. The % pointer is then incremented and the process repeated.

% ***** Run Loop Start ***** %

for i = 1:N,
pbminus = dl1(2,ptr(1));
upin2pt = [dl1(1,ptr(1)), dl2(1,ptr(2)), dl3(1, ptr(3)), dl4(1, ptr(4)),
dl5(1, ptr(5)), dl6(1, ptr(6))];
dnin2pt = [dl2(2,ptr(2)), dl3(2,ptr(3)), dl4(2, ptr(4)), dl5(2, ptr(5)),
dl6(2, ptr(6)), dl7(2, ptr(7))];

% ***** Boundary-Layer Loss Filter Calculations ***** %
[upin2pt(1),z_boundary(1,:)] = filter(b_boundary,a_boundary, upin2pt(1),z_boundary(1,:));
[pbminus,z_boundary(2,:)] = filter(b_boundary,a_boundary, pbminus,z_boundary(2,:));

% ******* Open-end Filter Calculations ******* %
[dl7(2,ptr(7)),z_open] = filter(b_open,a_open,dl7(1,ptr(7)),z_open);

% ***** First Tonehole Calculations ***** %
[temp1,zrup_th(1,:)] = filter(br_th(1,:),ar_th(1,:),upin2pt(1), zrup_th(1,:));
[temp2,ztdn_th(1,:)] = filter(bt_th(1,:),at_th(1,:),dnin2pt(1), ztdn_th(1,:));
dl1(2,ptr(1)) = temp1 + temp2;
[temp1,zrdn_th(1,:)] = filter(br_th(1,:),ar_th(1,:),dnin2pt(1), zrdn_th(1,:));
[temp2,ztup_th(1,:)] = filter(bt_th(1,:),at_th(1,:),upin2pt(1), ztup_th(1,:));
dl2(1,ptr(2)) = temp1 + temp2;

% ***** Second Tonehole Calculations ***** %
[temp1,zrup_th(2,:)] = filter(br_th(2,:),ar_th(2,:),upin2pt(2), zrup_th(2,:));
[temp2,ztdn_th(2,:)] = filter(bt_th(2,:),at_th(2,:),dnin2pt(2), ztdn_th(2,:));
dl2(2,ptr(2)) = temp1 + temp2;
[temp1,zrdn_th(2,:)] = filter(br_th(2,:),ar_th(2,:),dnin2pt(2), zrdn_th(2,:));
[temp2,ztup_th(2,:)] = filter(bt_th(2,:),at_th(2,:),upin2pt(2), ztup_th(2,:));
dl3(1,ptr(3)) = temp1 + temp2;

% ***** Third Tonehole Calculations ***** %
[temp1,zrup_th(3,:)] = filter(br_th(3,:),ar_th(3,:),upin2pt(3), zrup_th(3,:));
[temp2,ztdn_th(3,:)] = filter(bt_th(3,:),at_th(3,:),dnin2pt(3),
   ztdn_th(3,:));
dl3(2,ptr(3)) = temp1 + temp2;
[temp1,zrdn_th(3,:)] = filter(br_th(3,:),ar_th(3,:),dnin2pt(3),
   zrdn_th(3,:));
[temp2,ztup_th(3,:)] = filter(bt_th(3,:),at_th(3,:),upin2pt(3),
   ztup_th(3,:));
dl4(1,ptr(4)) = temp1 + temp2;

% ***** Fourth Tonehole Calculations ***** %

[temp1,zrup_th(4,:)] = filter(br_th(4,:),ar_th(4,:),upin2pt(4),
   zrup_th(4,:));
[temp2,ztdn_th(4,:)] = filter(bt_th(4,:),at_th(4,:),dnin2pt(4),
   ztdn_th(4,:));
dl4(2,ptr(4)) = temp1 + temp2;
[temp1,zrdn_th(4,:)] = filter(br_th(4,:),ar_th(4,:),dnin2pt(4),
   zrdn_th(4,:));
[temp2,ztup_th(4,:)] = filter(bt_th(4,:),at_th(4,:),upin2pt(4),
   ztup_th(4,:));
dl5(1,ptr(5)) = temp1 + temp2;

% ***** Fifth Tonehole Calculations ***** %

[temp1,zrup_th(5,:)] = filter(br_th(5,:),ar_th(5,:),upin2pt(5),
   zrup_th(5,:));
[temp2,ztdn_th(5,:)] = filter(bt_th(5,:),at_th(5,:),dnin2pt(5),
   ztdn_th(5,:));
dl5(2,ptr(5)) = temp1 + temp2;
[temp1,zrdn_th(5,:)] = filter(br_th(5,:),ar_th(5,:),dnin2pt(5),
   zrdn_th(5,:));
[temp2,ztup_th(5,:)] = filter(bt_th(5,:),at_th(5,:),upin2pt(5),
   ztup_th(5,:));
dl6(1,ptr(6)) = temp1 + temp2;

% ***** Sixth Tonehole Calculations ***** %

[temp1,zrup_th(6,:)] = filter(br_th(6,:),ar_th(6,:),upin2pt(6),
   zrup_th(6,:));
[temp2,ztdn_th(6,:)] = filter(bt_th(6,:),at_th(6,:),dnin2pt(6),
   ztdn_th(6,:));
```matlab
ztdn_th(6,:));
dl6(2,ptr(6)) = temp1 + temp2;
[temp1,zrdn_th(6,:)] = filter(br_th(6,:),ar_th(6,:),dnin2pt(6),
zrdn_th(6,:));
[temp2,ztup_th(6,:)] = filter(bt_th(6,:),at_th(6,:),upin2pt(6),
ztup_th(6,:));
dl7(1,ptr(7)) = temp1 + temp2;

% ******* Excitation Calculations ******* %
x = -(Zob*2*gama*z*l*alfa*sqrt((2/rho)) + sqrt(8*
(((Zob*gama*z*l*alfa)^2)/rho)-4*(1+ (4*psi*(gama*z*l.*alfa)
.^2)/Sr^2)*(2*pbminus - p_oc))/2*(1+(4*psi*(gama*z*l.*alfa).^2)
/Sr^2));

% ****** Pressure-Dependent Reflection Coefficient ****** %
refl = -0.7 + 0.4.*x;
if refl > 1
    refl = 1;
end;

dl1(1,ptr(1)) = 2*Zob*gama*z*l*alfa*x*sqrt(2/rho)./(1-refl+eps);

PB = [PB,pbminus];
DL = [DL,dl1(1,ptr(1))];

% The oral cavity is incremented until it reaches the desired value.
if p_oc < 1
    p_oc = p_oc + pinc;
end;

% ****** Increment Pointers & Check Limits ****** %
for j=1:length(L),
    ptr(j) = ptr(j) + 1;
    if ptr(j) > D(j),
```

ptr(j) = 1;
end;
end;
end;

env = [(0:5/N:1),ones(1,(4*N/5)-1)];
y = (PB+DL).*env;
norm_y = y/max(abs(y));

% *** Plot, Play and Wavwrite *** %
figure(1)
plot(real(norm_y), 'm')
title('Signal Output');
xlabel('Samples');
ylabel('Signal Level');

sound(real(norm_y),fs);
wavwrite(norm_y, fs, 16, filename);

SIG = abs(fft(norm_y,40000 ));
X = 20*log10(SIG);

figure(2)
plot(X(1:(length(X)/2)+1));
title('Signal Spectrum');
xlabel('Freq');
ylabel('dB');

Open pipe

function [b,a]=openpipe(nb,na,ra,fs,display)

%OPENPIPE Discrete filter least squares fit to Levine & Schwinger
% unflanged cylindrical bore reflectance.
%
% [B,A] = OPENPIPE(NB,NA,RA,FS,DISPLAY) gives real numerator and
denominator coefficients B and A of orders NB and NA respectively,
where RA is the radius of the cylinder (in meters) and FS is the
% desired sampling rate (in Hz). This function calls INVFREQZ with
% an omega^(-2) weighting function for the filter design process. A
% reference temperature of 26.85 degrees celsius (80 F) is assumed.
% If the value of DISPLAY is 1, the continuous-time and
% discrete-time reflectance magnitude and phase responses will be
% plotted.
% By Gary P. Scavone, CCRMA, Stanford University, 12 March 1997.

if nargin~=5,
  error('Number of arguments is incorrect.');
  return;
end;

% Physical constants and evaluation frequencies
n = 1024;  % Number of evaluation frequencies %
omega = pi/n:pi/n:pi;
c = 347.23;
k = omega*fs/c;

% The Levine & Schwinger results were calculated by numerical
% integrations out to the limit k*a <= 3.8. The following
% sixth-order polynomial approximations were then made to the
% reflectance magnitude and end correction terms. For values
% of k*a > 3.8, the reflectance magnitude and end corrections
% are approximated by smooth extensions.

rPoly = [-0.00121212308521, 0.01893693792170, -0.12135001867818,
         0.39739947149894, -0.61154450497445, 0.01320529396146,
         1.00000000000000];
loaPoly = [-0.00134446448269, 0.01444668268791, -0.06220104767324,
           0.136922621132, -0.16112693515620, -0.01536258568872,
           0.61000296711212];

z = k*ra;

if max(z) <= 3.8,
r = polyval(rPoly,z);
loa = polyval(loaPoly,z);
else
    i=1;
    while z(i) <= 3.8,
        i = i + 1;
    end;
    z1 = z(1:i-1);
    z2 = z(i:length(z));
    scale = polyval(rPoly,z(i))*z(i)^3;
    r = [polyval(rPoly,z1),scale./z2.^3];
    loa = polyval(loaPoly,z1);
    scale = polyval(loaPoly,3.8)*0.8^2;
    loa = [loa, scale./(z2 - 3).^2];
end;

R = -r.*exp(-2*j*z.*loa);

% Design digital filter using method described in JOS thesis,
% pp. 47-50 & 101-103.

% Weighting to help fit at low frequencies
wt = 1./omega.^2;
[b,a] = invfreqz(R,omega,nb,na,wt);

% Check filter stability %
ps = roots(a);
if length(ps)>0,
    for i=1:length(ps),
        if abs(ps(i)) >= 1.0,
            disp('Filter is unstable ... change design parameters!');
        end;
    end;
end;

% Plot responses %
if display==1,
    h = freqz(b,a,omega);
    clf

    subplot(2,1,1)
    plot(omega,abs(R),omega,abs(h));
    legend('Continuous-Time Response','Discrete-Time Response');
    title('Open Pipe Reflectance');
    ylabel('Magnitude');
    xlabel('Discrete-Time Radian Frequency');
    grid
    axis([0 pi 0 1.1])

    subplot(2,1,2)
    plot(omega,angle(R),omega,angle(h));
    ylabel('Phase (radians)');
    xlabel('Discrete-Time Radian Frequency');
    grid
    axis([0 pi 0 pi])
    disp('Paused ... hit any key to continue.');
    pause;
end;

Boundary

function [b,a, alpha]=boundary(nb,na,ra,fs,nz,display)

%BOUNDARY Discrete filter least squares fit to the attenuation % and phase delay characteristic of viscous and thermal % losses along the walls of a rigid cylindrical duct.
%
% [B,A] = BOUNDARY(NB,NA,RA,FS,NZ,DISPLAY) gives real numerator and % denominator coefficients B and A of order NB and NA respectively, % where RA is the duct radius (in meters), FS is the system % sampling rate (in Hz), and NZ is the number of unit sample delays % for which the boundary losses are modeled. This function calls % INVFREQZ with an omega^-2 weighting function for the filter % design process. A reference temperature of 26.85 degrees celsius % (80 F) is assumed. If the value of DISPLAY is 1, the continuous-
% time and discrete-time reflectance magnitude and phase responses
% will be plotted.
%
% By Gary P. Scavone, CCRMA, Stanford University, 12 March 1997.

if nargin~=6,
    error('Number of arguments is incorrect.');
    return;
end;

% Physical constants and evaluation frequencies
n = 1024; % Number of evaluation frequencies
omega = pi/n:pi/n:pi;
c = 347.23;
rho = 1.1769; % density of air
eta = 1.846*10^(-5); % viscosity of air
k = omega*fs/c;

% Duct characteristic impedance
Ro = rho*c/(pi*ra^2);

% Duct Losses (approximations for 300 K)
rv = ra*(rho*fs*omega/eta).^(0.5); % friction pg 42
alpha = k.*(1.045*rv.^(-1) + 1.08*rv.^(-2) + 0.75*rv.^(-3)); % attenuation coefficient per unit length
vpinv = (1 + 1.045*rv.^(-1))./c; % phase velocity

% Desired Filter Response
H = (exp(-j*omega.*(c.*vpinv - 1) - c*alpha/fs)).^nz; % pg144?

% Design digital filter using method described in JOS thesis,
% pp. 47-50 & 101-103.

% Weighting to help fit at low frequencies
wt = omega.^(-2);
[b,a] = invfreqz(H,omega,nb,na,wt);
% Check filter stability %

ps = roots(a);
if length(ps)>0,
  for i=1:length(ps),
    if abs(ps(i)) >= 1.0,
      disp('Filter is unstable ... change design parameters!');
    end;
  end;
end;
end;

% Plot responses %

if display==1,
  h = freqz(b,a,omega);
cf

subplot(2,1,1)
plot(omega,abs(H),omega,abs(h));
legend('Continuous-Time Response','Discrete-Time Response');
title('Boundary-Layer Losses');
ylabel('Magnitude');
xlabel('Discrete-Time Radian Frequency');
grid
axis([0 pi 0 1.1])

subplot(2,1,2)
plot(omega,angle(H),omega,angle(h));
ylabel('Phase (radians)');
xlabel('Discrete-Time Radian Frequency');
grid
axis([0 pi -pi 0])

%disp('Paused ... hit any key to continue.')%pause;
end;
function [br,ar,bt,at]=openhole(ra,rb,tw,rc,fs,display)

%OPENHOLE Discrete filter least squares fit to the open tonehole
% two-port reflectance and transmittance, as derived from
% Keefe (1990).
%
% [BR,AR,BT,AT] = OPENHOLE(RA,RB,TW,RC,FS,DISPLAY) gives real
% numerator and denominator reflectance coefficients BR and AR
% for a two-zero/one-pole filter, and transmittance coefficients
% BT and AT of order two, where RA is the main bore radius, RB is
% the tonehole radius, TW is the tonehole wall height, RC is the
% tonehole radius of curvature, and FS is the model sampling rate
% (in Hz). All length values should be given in meters. The
% discrete-time transmittance filter is designed using INVFREQZ
% with an omega^(-2) weighting function. The discrete-time
% reflectance filter is designed by Kopec's method, in conjunction
% with INVFREQZ. A reference temperature of 26.85 degrees celsius
% (80 F) is assumed. If the value of DISPLAY is 1, the continuous-
% time and discrete-time reflectance magnitude and phase responses
% will be plotted.
% RB can be a vector of tonehole radii, in which case the B's and
% A's are matrices of corresponding filter coefficients. Only the
% first tonehole reflectance responses will be plotted.
%
% By Gary P. Scavone, CCRMA, Stanford University, 12 March 1997.

if nargin~=6,
    error('Number of arguments is incorrect.');
    return;
end;

% Physical constants and evaluation frequencies

n = 1023;  % Number of evaluation frequencies
omega = pi/n*pi/n/pi;
c = 347.23;
rho = 1.1769;  % density of air
eta = 1.846*10^(-5);
\[ k = \omega * f_s / c; \]
\[ dv = \sqrt{2 * \eta / (\rho * \omega * f_s)}; \]

\% Bore and Tonehole characteristic impedances

\[ R_o = \rho * c / (\pi * r_a^2); \]
\[ R_h = \rho * c / (\pi * r_b^2); \]

\% Real part of propagation wavenumber in tonehole %

\[ \alpha = (3 * 10^{-5} / r_b) * \sqrt{\omega * f_s / (2 * \pi)}; \]

\% Tonehole geometric height

\[ t = t_w + (r_b / 8) * (r_b / r_a) * (1 + 0.172 * (r_b / r_a)^2); \]

\% Open tonehole effective length and specific resistance

\[ t_e = (\text{ones(size}(r_b')) * (1 / k) * \tan(t' * k) + r_b' * (1.4 - 0.58 * (r_b / r_a)^2) * \text{ones(size}(k))) / (1 - 0.61 * r_b' * k * \tan(t' * k)); \]
\[ x_i_e = 0.25 * (r_b'^2 + (t' * \text{ones(size}(k))) * \alpha + (\text{ones(size}(r_b'))) * (0.25 * k * dv)) * ((\log((2 / r_c) * r_b')) * \text{ones(size}(k))); \]

\% Open tonehole series equivalent length

\[ t_a = 0.47 * r_b * (r_b / r_a)^4 / (\tanh(1.84 * t / r_b) + 0.62 * (r_b / r_a)^2 + 0.64 * (r_b / r_a)); \]

\% Open Tonehole Shunt & Series Impedances %

\[ Z_{so} = R_h'^* \text{ones(size}(k)) * (j * \text{ones(size}(r_b')) * k * t_e + x_i_e); \]
\[ Z_{ao} = (-j * R_h * t_a'); * k; \]

\% Open Tonehole Reflectance & Transmittance %

\[ r_o_p = (Z_{so} * Z_{ao} - R_o^2) / (Z_{so} * Z_{ao} + 2 * R_o * Z_{so} + R_o^2); \]
\[ t_o_p = 2 * R_o * Z_{so} / (Z_{so} * Z_{ao} + 2 * R_o * Z_{so} + R_o^2); \]

\% The Matlab function invfreqz() doesn’t do a good job fitting % zeros for the reflectance. Thus, the filter is designed using
Kopec’s method (JOS thesis, pg. 46-47). A single pole is first fit to $\text{rop}(w)$ and the resulting response is divided out of $\text{rop}(w)$ to get $D_2(w)$. Two poles are then fit to $1./D_2(w)$. These poles become the zeros of the final discrete-time filter.

```matlab
na = 2; % poles
nb = 2; % zeros

% Weighting to help fit at low frequencies
wt = 1 ./ omega.^ 2;

br = zeros(length(rb),nb+1);
ar = zeros(length(rb),2);
btt = zeros(length(rb),nb+1);
at = zeros(length(rb),na+1);

for i=1:length(rb),
    [b1,a1] = invfreqz(rop(i,:),omega,0,1,wt);
temp = freqz(b1,a1,omega);
    [b2,a2] = invfreqz(temp./rop(i,:),omega,0,2,wt);
br(i,:) = b1*a2/b2;
ar(i,:) = a1;
br(i,:) = -abs(rop(i,1))*sum(ar(i,:))*br(i,:)/sum(br(i,:));
    [bt(i,:),at(i,:)] = invfreqz(top(i,:),omega,nb,na,wt);
end;

% Check filter stability
psr = roots(ar(i,:));
pst = roots(at(i,:));
if length(psr)>0,
    for e=1:length(psr),
        if abs(psr(e)) >= 1.0,
            disp('Filter is unstable...change design parameters!');
        end;
    end;
end;
if length(pst)>0,
    for e=1:length(pst),
        if abs(pst(e)) >= 1.0,
```

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disp('Filter is unstable ... change design parameters!');
end;
end;
end;
end;

% Plot responses %

if display==1,
h = freqz(br(1,:),ar(1,:),omega);
clf

subplot(2,1,1)
plot(omega,abs(rop(1,:)),omega,abs(h));
legend('Continuous-Time Response','Discrete-Time Response');
title('Open Tonehole Reflectance');
ylabel('Magnitude');
xlabel('Discrete-Time Radian Frequency');
grid
axis([0 pi 0 1.1])

subplot(2,1,2)
plot(omega,angle(rop(1,:)),omega,angle(h));
ylabel('Phase (radians)');
xlabel('Discrete-Time Radian Frequency');
grid
axis([0 pi -pi pi])

%disp('Paused ... hit any key to continue.')
%pause;
end;

Closed hole

function [br,ar,at,at]=closhole(ra,rb,tw,fs,display)

%CLOSHOLE Discrete filter least squares fit to the closed tonehole
% two-port reflectance and transmittance, as derived from
% Keefe (1990).
%
% [BR,AR,BT,AT] = CLOSHOLE(RA,RB,TW,FS,DISPLAY) gives real
numerator and denominator reflectance coefficients BR and AR and
transmittance coefficients BT and AT of order two, where RA is
the main bore radius, RB is the tonehole radius, TW is the
tonehole wall height, and FS is the model sampling rate (in Hz).
All length values should be given in meters. This function calls
INVFREQZ with a 1/omega^2 weighting function for the filter
design process. A reference temperature of 26.85 degrees celsius
(80 F) is assumed. If the value of DISPLAY is 1, the continuous-
time and discrete-time reflectance magnitude and phase responses
will be plotted.

RB can be a vector of tonehole radii, in which case the Bs and
As are matrices of corresponding filter coefficients. Only the
first tonehole reflectance responses will be plotted.

By Gary P. Scavone, CCRMA, Stanford University, 12 March 1997.

if nargin~=5,
error('Number of arguments is incorrect.');
return;
end;

% Physical constants and evaluation frequencies

n = 1024;  % Number of evaluation frequencies
omega = pi/n:pi/n:pi;
c = 347.23;
rho = 1.1769;
k = omega*fs/c;

% Bore and Tonehole characteristic impedances

Ro = rho*c/(pi*ra^2);
Rho = rho*c./(pi*rb.^2);

% Tonehole geometric height

t = tw + (rb/8).*(rb/ra).*(1 + 0.172*(rb/ra).^2);

% Closed tonehole series equivalent length
tac = 0.47*rb.*(rb/ra).^4./(coth(1.84*t./rb) + 0.62*(rb/ra) .^2 + 0.64*(rb/ra));

% Closed Tonehole Shunt & Series Impedances %

Zsc = -j*Rho'.* ones(size(k)) .* (cot(t'*k) + (t.*(0.25*(rb./t) .^2 + 0.58*(rb/ra).^2 - pi*rb ./ (4*t)))'*k);
Zac = (-j*Rho.*tac)'.* k;

% Closed Tonehole Reflectance %

rcl = (Zsc.*Zac - Ro^2) ./ (Zsc.*Zac + 2*Ro*Zsc + Ro^2);
tcl = 2*Ro*Zsc ./ (Zsc.*Zac + 2*Ro*Zsc + Ro^2);


na = 2;  % poles %
nb = 2;  % zeros %

% Weighting to help fit at low frequencies

wt = 1 ./ omega.^ 2;

br = zeros(length(rb),nb+1);
ar = zeros(length(rb),na+1);
bt = zeros(length(rb),nb+1);
at = zeros(length(rb),na+1);

for i=1:length(rb),
[br(i,:),ar(i,:)] = invfreqz(rcl(i,:),omega,nb,na,wt);
[bt(i,:),at(i,:)] = invfreqz(tcl(i,:),omega,na,na,wt);

% Check filter stability %

psr = roots(ar(i,:));
pst = roots(at(i,:));
if length(psr)>0,
for e=1:length(psr),
if abs(psr(e)) >= 1.0,
disp('Filter is unstable ... change design parameters!');
end;
end;
if length(pst)>0,
for e=1:length(pst),
    if abs(pst(e)) >= 1.0,
        disp('Filter is unstable ... change design parameters!');
    end;
end;
end;
end;

% Plot responses %
if display==1,
    h = freqz(br(1,:),ar(1,:),omega);
    clf
    subplot(2,1,1)
    plot(omega,abs(rcl(1,:)),omega,abs(h));
    legend('Continuous-Time Response','Discrete-Time Response');
    title('Closed Tonehole Reflectance');
    ylabel('Magnitude');
    xlabel('Discrete-Time Radian Frequency');
    grid
    axis([0 pi 0 1.1])

    subplot(2,1,2)
    plot(omega,angle(rcl(1,:)),omega,angle(h));
    ylabel('Phase (radians)');
    xlabel('Discrete-Time Radian Frequency');
    grid
    axis([0 pi -pi pi])
    disp('Paused ... hit any key to continue.')
    pause;
end;
Bibliography


