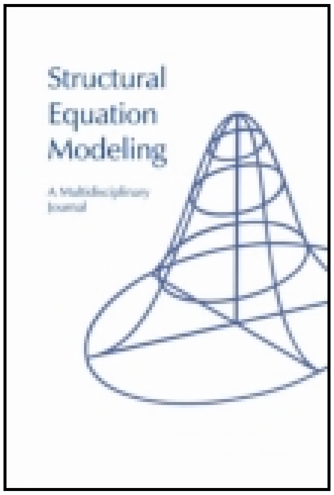

Structural Equation Modeling: A Multidisciplinary Journal
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/hsem20

A Confirmatory Factor Analysis Approach to Test Anxiety

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Published online: 05 Jun 2014.

Structural Equation Modeling: A Multidisciplinary Journal, 21: 455–467, 2014

ISSN: 1070-5511 print / 1532-8007 online
DOI: 10.1080/10705511.2014.915377

A Confirmatory Factor Analysis Approach to Test Anxiety

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This article addresses the role of test anxiety in aptitude testing. Our approach is rooted in confirmatory factor analysis (CFA). We find that the usual parameter constraints used for model identification in CFA have nontrivial implications for the effects of interest. We suggest 2 methods for dealing with this identification problem. First, we consider testable parameter constraints that identify the proposed model. Second, we consider structural relations that do not depend on model identification. In particular we derive the partial factor correlation between a test and an external variable, conditional on test anxiety, and show that this correlation (a) is not affected by the choice of model identification constraints, and (b) can be estimated using true score theory.

Keywords: confirmatory factor analysis, model identification, partial correlation, test anxiety

Test anxiety is anxiety that originates from the appraisal of being tested as threatening (Zeidner, 1998). In the modern-day context of high-stakes testing, it has become increasingly important to understand the relationship between test anxiety and test performance (e.g., Cizek & Burg, 2006; van der Embse & Hasson, 2012; Weems et al., 2010). In particular, it is important to (a) assess whether and to what extent test scores systematically underestimate the ability of anxious test takers, and (b) find a means of "correcting" test scores for the effects of test anxiety. In this article, we address these two issues by providing a measurement model of ability that incorporates test anxiety.

A related line of research concerns the criterion validity of intelligence tests (Reeve & Bonaccio, 2008; Reeve, Heggestad, & Lievens, 2009; Wicherts & Zand Scholten, 2010). It has been shown that test anxiety can bias the observed correlation between intelligence tests and their criteria, although the direction of the bias is a matter of ongoing debate. In general, this research highlights the importance of test anxiety as a confounding variable in correlation studies involving aptitude tests. We address this issue by modeling the structural relations among ability, test anxiety, and external variables.

Our approach is rooted in confirmatory factor analysis (CFA). We find that test anxiety presents unconventional challenges related to model identification and therefore leads to some relatively general methodological considerations. In particular, we argue that tests cannot be treated as unidimensional in the presence of test anxiety, but must be regarded as (possibly) two-dimensional. Unlike similar multidimensional models (e.g., the bifactor model, multitrait–multimethod models), it cannot be assumed that ability and test anxiety are orthogonal. Consequently we are faced with a specific type of rotational indeterminacy, which we refer to as *confound indeterminacy.*

We propose two main ways of dealing with confound indeterminacy. First, we consider testable parameter constraints that serve to eliminate it. Depending on the research context, these constraints may be viewed either as simplifying assumptions placed on the model or as substantive hypotheses. Once the model is identified, the contributions of ability and test anxiety to test performance can be disentangled, leading to a measure of ability that is not biased by test anxiety.

Second, we consider structural relationships that remain invariant under confound indeterminacy. Specifically, we

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show that the partial factor correlation between ability and an external variable, conditional on test anxiety, is invariant and can be estimated using the methods of true score theory. This result is useful for researchers who are committed to using observed correlations, but it does not allow for interpretation of other structural relationships in the model.

In contrast to recent approaches (e.g., da-Silva & Gomes, 2011; Wicherts & Zand Scholten, 2010), we focus on the role of test anxiety as a continuous latent variable rather than its role as a manifest grouping variable. The use of a grouping variable leads naturally to considerations about measurement invariance and differential item functioning. The use of a continuous factor allows us to address these same issues, as well as structural relations among tests, test anxiety, and other variables. Moreover, our approach is congenial to the growing literature on the assessment of test anxiety via self-report questionnaires (e.g., Cizek & Burg, 2006; Oostdam & Meijer, 2003; Wren & Benson, 2004). In keeping with the usual practice of CFA, we phrase the discussion in terms of continuous indicators (e.g., test scores) that are linearly related to their measurement constructs. Extension to discrete indicators with nonlinear measurement equations (e.g., item level analysis) can be made by means of the familiar arguments (e.g., McCullagh & Nelder, 1989; Takane & de Leeuw, 1987). We do not consider nonlinear structural relations (e.g., latent moderation).

It should be emphasized that this article presents a methodological solution to a substantive problem. It is therefore neither exclusively methodological nor exclusively applied in its scope. Because the problem motivates novel research in CFA, it will be of interest to methodologists. However, we have not attempted to present our results in anything like their highest generality, which is a task for further research. Our presentation is specific to the context of test anxiety, and we illustrate the application of our results using an example from educational testing.

The remainder of this article is organized as follows. The next section reviews the literature on test anxiety to develop an appropriate CFA model. The subsequent section demonstrates that the proposed model is subject to confound indeterminacy and considers parameter constraints that eliminate this indeterminacy. Next we address the partial factor correlation between a test and an external variable, conditional on test anxiety. The penultimate section contains the example. We conclude by discussing the significance of these results for research on test anxiety and for applications of CFA that face similar types of rotational indeterminacy.

A CFA MODEL OF TEST ANXIETY

Test anxiety is again receiving more attention in practical settings, especially with relevance for high-stakes testing in the the school context (e.g., Cizek & Burg, 2006; van der Embse & Hasson, 2012; Meijer & Oostdam, 2007; Parks-Stamm, Gollwitzer, & Oettingen, 2007; Putwain, 2007; Putwain & Daniels, 2010; Weems et al., 2010), and in the personnel selection context (e.g., Dobson, 2000; McCarthy & Goffin, 2005; Proost, Derous, Schreurs, Hagtvet, & Witte, 2008). Test anxiety also has theoretical importance, for instance when the relationship between personality and intelligence is at stake (e.g., Furham, Forde, & Cotter, 1998; Moutafi, Furnham, & Tsaousis, 2006; Zeidner, 1995), and in explaining group differences in test performance (e.g., Araki, 1992; di Maria & di Nuovo, 1990; Zeidner & Safir, 1989). In this section we provide a selective review of this literature with the purpose of developing a general model that incorporates the substantive theory. We first discuss the role of test anxiety in the measurement of ability and then the relationships among test anxiety, test scores, and external variables.

Test Anxiety and the Measurement of Ability

It is well established that test anxiety is negatively correlated with aptitude test scores (Ackerman & Heggestad, 1997; Hembree, 1988; Zeidner, 1998). Oostdam and Meijer (2003) considered two main hypotheses to explain this correlation. The *deficit hypothesis* states that test takers with a skill deficit (i.e., lower ability) experience higher anxiety. In its strongest phrasing, the hypothesized skill deficit fully explains the observed correlation between test scores and test anxiety. Several authors have argued for a variation of this hypothesis (e.g., Ball, 1995; Pekrun, 1992; Tobias, 1992).

In terms of CFA, we interpret the deficit hypothesis to mean that ability and test anxiety are correlated. This is depicted in Figure 1 by the factor correlation (ϕ_{AX}). The circles labeled θ_X and θ_A denote ability and test anxiety,

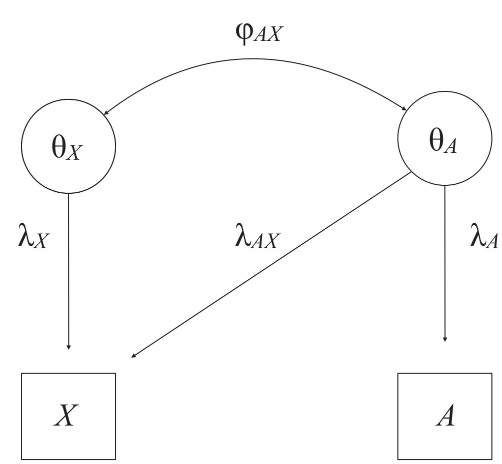

FIGURE 1 Two possible explanations of the correlation between test scores (X) and measures of test anxiety (A). Both X and A may represent multiple indicators. The aptitude or ability being measured is denoted as θ_X and test anxiety is denoted by θ_A. The correlation λ_{AX} denotes a deficit effect. The cross-loading λ_{AX} denotes an interference effect. The arrows λ_X and λ_A are the factor loadings of the indicator variables on their measurement constructs. Error terms are suppressed for visual clarity.

respectively. The squares labeled X and A denote their indicator variable(s). We do not specify whether ability causes test anxiety or vice versa, nor do we eliminate the possibility that they are related through some third variable. Such considerations are of central importance in the test anxiety literature (see McDonald, 2001), but they take us beyond the scope of measuring ability.

The second explanation considered by Oostdam and Meijer (2003) is the *interference hypothesis.* Under this hypothesis, test anxiety prevents anxious test takers from performing at their true level of ability. They provided support for this hypothesis by showing that students with higher test anxiety perform better in less stressful situations. Several other studies have also provided evidence for the interference hypothesis. Hembree (1988) listed a large number of anxiety-reducing interventions that yield improvements in performance for anxious test takers. Parks-Stamm et al. (2007) found an interaction between test anxiety and test-taking strategy. Students with high test anxiety perform better with a temptation-inhibiting strategy (staying away from distractions) than with a task-facilitating strategy (intensifying one's efforts on the test), whereas the effect is opposite and much smaller for students with low test anxiety. Their result again shows that the relationship between test scores and test anxiety can be manipulated independent of ability, which supports the interference hypothesis.

In Figure 1 we represent the interference hypothesis by letting test scores load on test anxiety (λ_{AX}). This means that test scores are not conditionally independent given ability, or equivalently, that the test is not a reliable measure of ability. The presence of a nonzero cross-loading has serious implications for educational practice (Cizek & Burg, 2006).

These considerations lead us to a two-dimensional model of test scores in which the factors, ability and test anxiety, are correlated. A similar model was arrived at by Wicherts and Zand Scholten (2010). The model has the following advantages. First, it disentangles the contributions of ability and test anxiety to test performance. These contributions are represented by the factor loadings λ_X and λ_{AX}, respectively. Second, even though test scores (X) might be confounded by test anxiety (θ_Y), the resulting measure of ability (θ_X) is not. In other words, we obtain a reliable measure of ability for anxious test takers. Additionally, the model allows us to test hypotheses about test anxiety within a CFA framework.

Unfortunately, the model in Figure 1 is subject to rotational indeterminacy. Although this will be obvious to the experienced factor analyst, it is perhaps less obvious that the usual identification constraints employed in CFA have nontrivial implications for the effects of interest. Identification of the model is discussed in the following section.

The model presented in Figure 1 does not exhaust the possible explanations of the correlation between test scores and test anxiety. For example, a direct effect of X on A was described by Deffenbacher (1978). This effect was induced by a particular experimental situation that is not reflective of usual testing practices, so we do not address it here. Zeidner (1995) discussed the possibility that measures of test anxiety could be confounded by cognitive ability, with less able students perceiving themselves as having higher anxiety. We are not aware of any evidence to support this possibility and therefore do not consider it further. We also note that both ability and test anxiety can be treated as multidimensional. Several authors have discussed this kind of approach to test anxiety (e.g., Meijer & Oostdam, 2007; Wren & Benson, 2004), and multidimensional models of ability are well known (e.g., Carroll, 1993). It is then possible to include multiple latent variables for both ability and test anxiety, with a structure similar to that in Figure 1 being possible for each pairwise matching of the two types of latent variables. Despite these limitations, our view is that any serious attempt to model the role of test anxiety in the measurement of ability must minimally include the effects depicted in Figure 1.

Test Anxiety, Test Scores, and External Variables

The deficit and interference effects described earlier also have consequences for correlation studies involving aptitude tests. In the context of validity studies, evident external variables are school grades (e.g., Hembree, 1988; Pintrich & Groot, 1990) and job performance (e.g. Bertua, Anderson, & Salgado, 2005; Hunter & Hunter, 1984; Salgado & Anderson, 2003). The relationship of test anxiety with school grades is negative, and the direction of the relationship with job performance is unclear. Both school grades and job performance are often evaluated in a testing context, in which case the foregoing considerations about measurement are again relevant.

To capture the relationship between a test and a single external variable (θ_Y), we extend the model in Figure 1 as shown in Figure 2. The correlation between ability and the external variable is denoted ϕ_{XY}. The paths labelled ϕ_{AY} and λ_{AY} are the deficit and interference effects of test anxiety on the external variable, respectively. These might or might not be applicable in a given context. We refer to the submodel corresponding to Figure 1 as the AT (anxiety–test) model. The corresponding effects running between test anxiety and the external variable we refer to as the AE (anxiety–external variable) model. The complete model includes the AT submodel, the AE submodel, and the correlation between θ_X and θ_Y.

Although it is common practice to use the manifest correlation between X and Y to measure the association between ability and external variables, this can hardly be recommended. Wicherts and Zand Scholten (2010) showed that manifest correlations are biased in the presence of interference effects. It is therefore preferable to take a latent variable approach. Nonetheless, we show that the partial correlation between X and Y, controlling for A, is not affected by interference effects when using the usual correction for

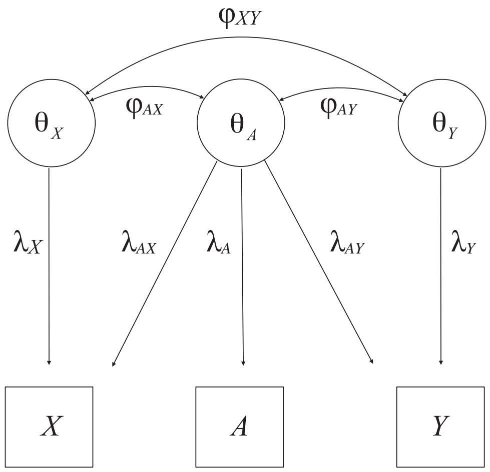

FIGURE 2 Extension of the measurement model in Figure 1 to include a third variable of interest, θ_Y, with indicator(s) Y and factor loading(s) λ_Y. The paths labeled ϕ_{AY} and λ_{AY} mirror the deficit and interference effects described in Figure 1. The correlation ϕ_{XY} represents the relationship between ability and the external variable. The remaining paths are described in Figure 1. Error terms are suppressed for visual clarity.

attenuation. This result is useful for researchers who are committed to using observed correlations.

Summary

This section has selectively reviewed the literature on test anxiety to develop an appropriate CFA model. The model requires that test scores, and possibly external variables, are treated as two-dimensional with correlated factors. In the following section we discuss the identification of this model.

CONFOUND INDETERMINACY IN THE AT MODEL

This section focuses exclusively on the AT model depicted in Figure 1. Entirely similar considerations come into play when addressing the AE model, so that we can generalize to the full model depicted in Figure 2 without needing to explicitly consider both submodels. We begin by showing that the AT model is not identified, and in particular that it is subject to a specific type of rotational indeterminacy, which we refer to as confound indeterminacy. Following the usual practice in CFA, the model would be identified by imposing a simple structure on a subset of the indicator variables or requiring an orthogonal factor solution (see Millsap, 2001). We show that these constraints render the effects of interest uninterpretable. We therefore consider alternative methods of identifying the model.

As noted, the identification of the AT model is related to the topic of rotational indeterminacy. For reference we provide a brief review of rotational indeterminacy here. The basic idea is that, in the unrestricted multidimensional factor model, factor scores and factor loadings can be linearly transformed without changing the model-implied covariance matrix (see Millsap, 2001; Yanai & Ichikawa, 2007). Therefore rotational indeterminacy is a type of model unidentification—numerically different parameter arrangements correspond to numerically identical observed moments, and hence numerically identical goodness of fit. To deal with rotational indeterminacy, some of the model parameters must be set to fixed values. In exploratory factor analysis, this is accomplished by means of various analytic factor rotations (see Browne, 2001, for a review). In CFA, structural properties are imposed on the factor pattern and the factor covariance matrix until sufficient constraints have been given to eliminate rotational indeterminacy. As we now discuss, the factor pattern implied by the AT model is not sufficiently constrained to eliminate rotational indeterminacy.

Rotational Indeterminacy in the AT Model

There are numerous ways to establish the (un)identification of models for covariance structures (e.g., Bekker, Merckens, & Wansbeek, 1989; Bollen, 1989; Bollen & Bauldry, 2010). The case presented here allows for a simple algebraic approach. We show that the AT model is subject to rotational indeterminacy by providing a general form for the rotation matrix. This has the advantage of allowing us to explicitly consider what restrictions will serve to identify the model and to describe the consequences of those restrictions for the effects of interest. We assume throughout that the scales of the latent variables are fixed by setting their means to zero and their variances to one. As is usual, we treat rescaling of the latent variables as a trivial (i.e., ignorable) form of rotational indeterminacy.

The matrix of factor loadings corresponding to Figure 1 is

$$\Lambda = \begin{bmatrix} \lambda_{X_1} & \lambda_{AX_1} \\ \lambda_{X_2} & \lambda_{AX_2} \\ \vdots & \vdots \\ \lambda_{X_J} & \lambda_{AX_J} \\ 0 & \lambda_{A_1} \\ 0 & \lambda_{A_2} \\ \vdots & \vdots \\ 0 & \lambda_{A_K} \end{bmatrix} \tag{1}$$

with $j = 1, \ldots, J$ indexing the indicators of θ_X and $k = 1, \ldots, K$ indexing the indicators of θ_A. The loadings λ_{X_j} and λ_{A_k} correspond to the measurement models of θ_X and θ_A, respectively, and we assume that these are nonzero. The cross-loadings λ_{AX_j} denote the interference effects of θ_A on the X_j. The zeros are a structural restriction given by the assumption that indictors of test anxiety do not load on ability. In keeping with Figure 1, the factor correlation is a free parameter that represents the deficit effect. In this section we denote the factor correlation as ϕ.

Rotational indeterminacy of Λ can be demonstrated by showing the existence of a nonsingular rotation matrix R such that ΛR contains the same structural zeros as Λ. It can be verified that any such rotation matrix has the form

$$\mathrm{R} = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \tag{2}$$

where *a, b, c* are real numbers with $a \neq 0 \neq c$ determining the scale of the factors. Setting $b = 0$ identifies the model up to the scale of the factors. This was established using the linear algebra package of Maple 10 (Maplesoft, 2005) to evaluate the Wald rank rule for the transformed model (see Bollen & Bauldry, 2010). Therefore rotational indeterminacy is the only source of unidentification in the AT model, and nontrivial rotational indeterminacy occurs only when $b \neq 0$. We refer to this type of indeterminacy as confound indeterminacy.

To eliminate confound indeterminacy, we require additional constraints on the AT model that are violated whenever $b \neq 0$. The possible constraints can be determined by computing the rotation. Computation of ΛR shows that b appears in the cross-loadings, λ_{AX_j}, but not the other factor loadings. The rotated values are

$$\lambda^*_{AX_j} = b\,\lambda_{X_j} + c\,\lambda_{AX_j} \tag{3}$$

for $j = 1, \ldots, J$. The rotated factor correlation can be written

$$\phi^* = \frac{\phi - b/c}{\sqrt{(b/c)^2 - 2\phi\, b/c + 1}}. \tag{4}$$

Equations 3 and 4 show that to eliminate confound indeterminacy, it is minimally required that one of the interference effects or the deficit effect be fixed. For example, if we require that $\lambda^*_{AX_j} = c\,\lambda_{AX_j} = r$, for fixed values of j and $r \in \mathbb{R}$, this requirement can only be satisfied if $b = 0$. Therefore, fixing one of the interference effects serves to identify the model. Similarly, requiring $\phi^* = \phi = s$ for a fixed value of $s \in (-1, 1)$ also identifies the model.

In the usual practice of CFA, we would identify the AT model by setting either r or s to an arbitrary numerical value, and the value chosen is most commonly zero. Setting $r = 0$ is equivalent to forcing a simple structure on a subset of the factor pattern; setting $s = 0$ is equivalent to requiring an orthogonal factor solution. Thus it appears that we must "give up" one of the effects of interest to identify the model. However, the situation is far worse than this. As we show in the next subsection, selecting an arbitrary value for the deficit effect also implies that the numerical values of the interference effects are arbitrary. A similar situation holds when fixing one of the interference effects. In short, if we identify the AT model using the usual parameter constraints employed in CFA, none of the effects of interest are interpretable.

Consequences of Usual CFA Identification Constraints for the AT Model

In the following we describe the consequences of identification constraints placed on the AT model in terms of factor rotations applied to an unknown data-generating model. Model parameters without superscripts (e.g., ϕ, λ_{X_j}) denote the data-generating parameters. These can be thought of as the true parameters in the population of interest. Identification constraints are described in terms of factor rotations applied to the data-generating parameters. Quantities relating to rotations are denoted by Latin letters (e.g., R, r) and are due to identification choices made by the researcher. Model parameters with the asterisk superscript (e.g., ϕ^*, $\lambda^*_{X_j}$) denote the parameter values of an identified model. These are regarded as functions of the true parameter values and the chosen identification constraints.

Consider first the implications of fixing the interference effect on X_j to some arbitrary value $r \in \mathbb{R}$, where j and r are according to the convenience of the researcher. This can be interpreted as choosing a rotation matrix R such that $\lambda^*_{AX_j} = r$. Inserting this restriction into Equation 3 implies that

$$b = \left(r - c\lambda_{AX_j}\right)/\lambda_{X_j}. \tag{5}$$

Note that setting $r = 0$, which would be the usual approach, does not imply that $b = 0$. Thus the identification constraint represents a nontrivial rotation of the data-generating parameters. To have $b = 0$, we must have $r = c\lambda_{AX_j}$. This means that the researcher would have to guess the true interference effect to identify the model in this manner.

We can compute the value of the rotated factor correlation implied by Equation 5 using Equation 4:

$$\phi^* = \frac{\phi - (r/c - \lambda_{AX_j})/\lambda_{X_j}}{\sqrt{(\phi - (r/c - \lambda_{AX_j})/\lambda_{X_j})^2 - \phi^2 + 1}}. \tag{6}$$

It is a complicated function involving both the value chosen for the constraint, r, and the data generating parameters, λ_{AX_j}. Figure 3 shows Equation 6 for $\phi = 0$, $r = 0$, and various combinations of λ_{AX_j} and λ_{X_j}. This corresponds to a case in which there is no deficit effect in the population ($\phi = 0$), and the model is identified by setting one of the measurement interference effects to zero ($r = 0$). Figure 3 shows how the model-implied deficit effect, ϕ^*, departs from the true value of zero. Specifically, it is seen that ϕ^* is a nonlinear function of the true measurement interference effect, λ_{AX_j}, and that this relationship is moderated by λ_{X_j}.

In general, identifying the AT model by fixing one of the interference effects to an arbitrary value will result in a

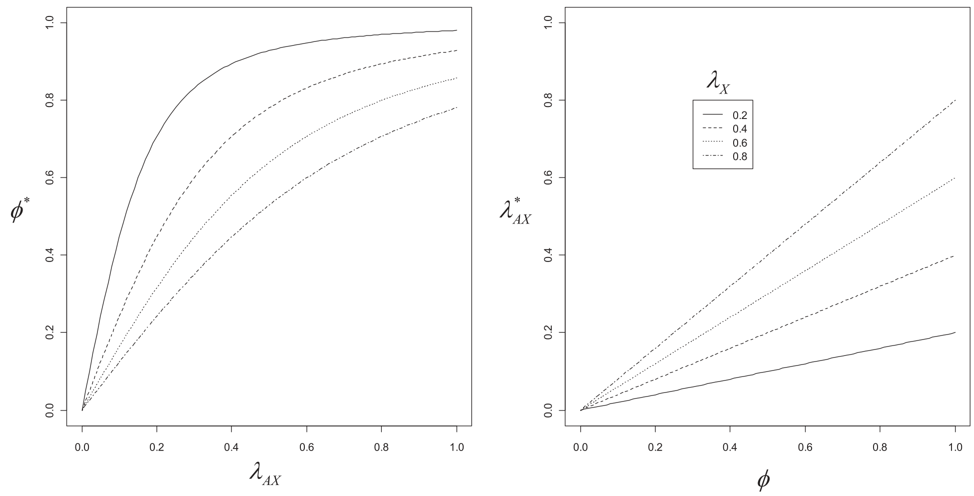

FIGURE 3 Confounding of null effects by model identification constraints. The left panel shows Equation 6 as a function of λ_{AX} and λ_X with $r = 0$ and $\phi = 0$. Because it is the ratio λ_{AX}/λ_X that determines the magnitude of ϕ^*, the raw values of the loadings are not relevant. The right panel shows Equation 7 as a function of ϕ and λ_X with $s = 0$ and $\lambda_{AX} = 0$. Here $c = 1$, so that the magnitude of the factor loadings is interpreted relative to the variance of θ_A, which is equal to 1. Because ϕ^* and λ^*_{AX} are both odd functions, only positive values of the arguments are shown. Values of λ_X are given in the right panel.

model-implied deficit effect that is not equal to the deficit effect in the population of interest. In research applications, this means that the numerical value of the estimated deficit effect is uninterpretable. In particular, testing hypotheses about the value of ϕ^* does not allow one to make conclusions about ϕ.

Fixing one of the interference effects also has consequences for the remaining interference effects. Substituting Equation 5 into Equation 3, we have

$$\lambda^*_{AX_k} = c\lambda_{AX_k} - (r - c\lambda_{AX_j}\lambda_{A_k})\big/\lambda_{X_j} \quad j \neq k. \tag{7}$$

Therefore, when one interference effect is fixed, the remaining interference effects are a function of the value to which the effect is fixed, r, and the value of the data-generating parameter, λ_{AX_j}, as well as other quantities. The functional relationship here is similar to that considered directly, so we do not discuss it further.

We now address the case where the model is identified by fixing the value of the factor correlation. This is equivalent to choosing a rotation such that $\phi^* = s$, for some arbitrary fixed value $s \in (-1, 1)$. Then Equation 4 implies

$$b\big/c = \sqrt{\frac{s^2(1-\phi^2)}{1-s^2}} - \phi \tag{8}$$

and substitution of Equation 8 into Equation 3 shows that

$$\lambda^*_{AX_j}\big/c = \lambda_{AX_j} + \left(\sqrt{\frac{s^2(1-\phi^2)}{1-s^2}} - \phi\right)\lambda_{X_j} \tag{9}$$

for $j = 1, \ldots, J$. It can be verified $\lambda^*_{AX_j}/c = \lambda_{AX_j}$ only when $\phi = s$. Thus we arrive at a situation similar to that described earlier: Identifying the AT model by fixing the deficit effect to an arbitrary value results in model-implied interference effects that are not, in general, equal to the interference effects in the population of interest.

The right panel of Figure 3 plots Equation 9 for $\lambda_{AX_j} = 0$, $s = 0$, and $c = 1$. This corresponds to a case in which there is no interference effect on variable X_j in the population of interest and the model is identified by an orthogonal factor solution. The model-implied interference effect, $\lambda^*_{AX_j}$, is seen to be a linear function of the true deficit effect, ϕ, and this is again moderated by the reliability of X_j.

The results of this subsection can be conveniently summarized in terms of a trade-off between the interference effects and the deficit effect: Identification of the model requires fixing one, but doing so influences the numerical value of the other. Consequently, none of the effects of interest can be interpreted if the model is identified by arbitrary parameter constraints. In applications, this means that the numerical values of the parameter estimates and their related significance tests would be strictly meaningless. Therefore we turn to consider alternative means of identifying the AT model.

Testable Identification Constraints on the AT Model

The foregoing has made it clear that nonarbitrary identification constraints are required to interpret the AT model. The strategy we pursue here is to introduce more constraints than are necessary for identification, and in particular we consider equality restrictions among the interference effects. These constraints can be seen as simplifying assumptions or can be motivated by substantive hypotheses about test anxiety. The resulting models are properly nested within the just-identified AT model, so the validity of the constraints can be tested via the usual procedures for nested models (e.g., Bollen, 1989; Satorra & Bentler, 2001). Thus we replace the problem of identification with that of testing hypotheses about the interference effects.

The proposed solution does come with disadvantages. Although the constraints we consider serve to formally identify the model, there exist cases where the model might be numerically unidentified due to specific configurations of the ability loadings. We describe these configurations in Equation 11. Additionally, it might turn out that none of the available constraints is compatible with a given data set (i.e., the nested hypotheses are all rejected). Although it is not ideal, our approach to identification does allow test anxiety to be modeled within a CFA framework; this is a distinct advantage over the use of arbitrary identification constraints.

Using the notation of the previous section, equality constraints on the interference effects require setting $\lambda^*_{AX_j} = \lambda_{AX_k}$ for some choices of $j \neq k$. Substituting this requirement into Equation 5 shows that

$$b = (\lambda_{AX_k} - c\lambda_{AX_j})\big/\lambda_{A_j}. \tag{10}$$

Assuming that $c = 1$, then $b = 0$ just in case $\lambda_{AX_j} = \lambda_{AX_k}$. In other words, if it is true that $\lambda_{AX_j} = \lambda_{AX_k}$ in the population of interest, then using this equality constraint to identify the model implies that $b = 0$, and therefore that we can interpret the effects of interest. Accordingly, we require a test of the hypothesis that $\lambda_{AX_j} = \lambda_{AX_k}$.

A single equality constraint on the interference effects is only sufficient to just identify the AT model. Therefore we cannot test the single constraint. Imposing multiple equality constraints across the interference effects, however, does yield a testable hypothesis in the form of a properly nested model. For example, we could hypothesize that the interference effect is constant over all $j = 1, \ldots, J$ ability indicators, resulting in a total of $J - 1$ parameter restrictions on the AT model. This could be referred to as a tau-equivalent model for the interference effects. To obtain the nesting model, the AT model can be just identified by imposing any one of the constraints already discussed. Assuming that the AT model contains the data-generating process, the likelihood ratio statistic has an asymptotic chi-square distribution on $J - 2$ degrees of freedom under the null hypothesis.

In CFA it is usual to impose tau-equivalence as a numerical simplification, without having a specific empirical motivation outside of the fact that the data are congenial to the simplification. In the context of test anxiety, tau-equivalence of the interference effects might also represent a substantive hypothesis about test anxiety. For example, if the ability indicators are different test items on a single test, tau-equivalence means that each item is equally anxiety provoking, regardless of its content or its position within the test. Such a hypothesis was addressed by Oostdam and Meijer (2003). If the ability indicators are different testing situations, tau-equivalence means that test takers experience equal amounts of test anxiety regardless of the situation. On the other hand, it is commonly supposed that test anxiety should be increased in high-stakes testing as compared to low-stakes testing (e.g., Cizek & Burg, 2006). This supposition can also be tested under the AT model. For example, we could have test takers write a series of examinations (or exam items) in either a high-stakes or low-stakes situation and test whether tau-equivalence holds within but not between the two situations. Further equality constraints can be adduced from the test anxiety literature. We provide an illustration of this approach to model identification in our example.

Before moving on, we point out an important limitation. Consider the following two interference effects from Equation 3 obtained by using the equality $\lambda_{AX_j} = \lambda_{AX_k}$:

$$\begin{aligned} \lambda^*_{AX_j} &= b\lambda_{X_j} + c\,\lambda_{AX_k} \qquad j \neq k \qquad \text{and} \\ \lambda^*_{AX_k} &= b\lambda_{X_k} + c\,\lambda_{AX_k}. \end{aligned} \tag{11}$$

Equations 11 show that there are in fact two cases in which the rotated factor loadings retain the equality constraint (i.e., $\lambda^*_{AX_j} = \lambda^*_{AX_k}$). The case we described earlier occurs when $b = 0$. The second case is when $\lambda_{X_j} = \lambda_{X_k}$; that is, when tau-equivalence also holds for the ability indicators. In this case, the equality restriction in Equation 10 does not identify the model. In practice, this is recognizable by nonconvergence of estimation algorithms and by obtaining different estimates from disparate starting values. In some situations this can be remedied by different choices of *j, k*, or both. However, if a tau-equivalent model holds for the ability indicators, the identification strategy proposed here will no longer work.

Summary

The general message to be taken from this section is that the usual parameter constraints employed in CFA are not tenable for modeling test anxiety. To deal with this problem we have discussed some nontrivial (i.e., testable) parameter constraints that serve to identify the AT model. These constraints readily correspond to substantive hypotheses about test anxiety, and therefore provide an appropriate solution to the identification problem in this context.

As noted at the outset of this section, entirely similar considerations apply to the AE model shown on the right side of Figure 3. Therefore two sets of constraints of the kind discussed here must be imposed to identify the full model. Given these constraints, the correlation between ability and an external variable can be estimated, in addition to the deficit and interference effects. We illustrate this in our example.

As noted, the constraints we have discussed are not effective when the ability indicators are tau-equivalent. Moreover, full-blown structural equation modeling is not a viable option in many correlation studies. In such cases it is desirable to have an alternative measure of association for test scores that is not confounded by test anxiety. This is the topic of the following section.

A STRUCTURAL RELATION THAT IS INVARIANT UNDER CONFOUND INDETERMINACY

In this section we return to the full model depicted in Figure 2. We show that the partial correlation between θ_X and an external variable θ_Y, given θ_X, is invariant under confound indeterminacy. This means that the partial correlation provides a measure of association between θ_X and θ_Y that is interpretable regardless of the identification constraints employed. We also derive the partial correlation between X and Y. This is a biased estimator of the latent partial correlation, but the usual correction for attenuation serves to correct this bias. Thus it is possible to estimate the partial correlation between θ_X and θ_Y without estimating the CFA model.

From a methodological perspective, this is a curious result that presents interesting possibilities for generalization. From the perspective of correlational research involving aptitude tests, it means that we can control for the confounding effects of test anxiety without having to directly model those effects. Of course, the applicability of the partial correlation will depend on the research question at hand.

Let

$$\mathrm{f}=\begin{bmatrix}\theta_X\\ \theta_Y\\ \theta_A\end{bmatrix}\quad \Phi=\begin{bmatrix}1 & - & -\\ \phi_{XY} & 1 & -\\ \phi_{AX} & \phi_{AY} & 1\end{bmatrix}\quad \mathrm{L}=\begin{bmatrix}\lambda_X & 0 & \lambda_{AX}\\ 0 & \lambda_Y & \lambda_{AY}\\ 0 & 0 & \lambda_A\end{bmatrix} \tag{12}$$

describe the full model in Figure 2. The vector f contains the factors and Φ is their correlation matrix. L gives an abbreviated factor pattern. Each row of the complete factor pattern must have structural zeros corresponding to one of the three rows shown in L. Again we assume that the scales of the factors are fixed by setting their means to zero and variances to one.

As with the AT and AE submodels, the full model is subject to confound indeterminacy. The rotation matrix for the factor pattern, T, and the inverse rotation that transforms the factors, T^{-1}, have the form

$$\mathrm{T}=\begin{bmatrix}t_{\mathrm{X}} & 0 & t_{AX}\\ 0 & t_{\mathrm{Y}} & t_{AY}\\ 0 & 0 & t_A\end{bmatrix}\quad \mathrm{T}^{-1}=\begin{bmatrix}\frac{1}{t_X} & 0 & \frac{-t_{AX}}{t_X t_A}\\ 0 & \frac{1}{t_Y} & \frac{-t_{AY}}{t_Y t_A}\\ 0 & 0 & \frac{1}{t_{\mathrm{A}}}\end{bmatrix} \tag{13}$$

The null entries are implied by the structural zeros in the factor pattern, as is readily verified. The other values are assumed to be nonzero real numbers. For later reference we note that T, T^{-1}, L, and L^{-1} all have the same structural zeros. This means that the abbreviated factor pattern and its inverse are admissible rotations under confound indeterminacy.

Let $\phi_{XY|A}$ denote the partial correlation between θ_X and θ_Y conditional on θ_A Also let $\phi^*_{XY|A}$ denote the partial correlation resulting from application of the rotation T to the model. In the Appendix we show that

$$\phi^*_{XY|A} = \phi_{XY|A}. \tag{14}$$

Equation 14 says that the partial correlation of θ_X and θ_Y, given θ_A, is invariant under confound indeterminacy. Therefore the value of $\phi_{XY|A}$ is not affected by the constraints used to identify the model. The next section provides an empirical illustration of this fact. We note that the other partial correlations that can be computed between the factors are affected by confound indeterminacy (see Appendix).

We now consider the manifest partial correlation between an aptitude test X and an external variable Y when controlling for an anxiety indicator A. The zero-order covariance matrix of the three indicators can be written as $\Sigma = \mathrm{L}\Phi\mathrm{L}' + \Psi$, where L and Φ are from Equation 12 and Ψ is the diagonal matrix of error variances. Let $\Sigma^* = \Sigma - \Psi = \mathrm{L}\Phi\mathrm{L}'$ and let $\rho^*_{XY|A}$ denote the partial correlation between X and Y computed on Σ^*.

As noted in connection with Equation 13, L^{-1} is in fact an admissible rotation under confound indeterminacy. Thus we can write $\mathrm{T} = \mathrm{L}^{-1}$ so that $\Sigma^* = \mathrm{T}^{-1}\Phi\mathrm{T}^{-1'}$ is seen to be a rotation of the factor correlation matrix. Then Equation 14 implies

$$\rho^*_{YX|A} = \phi_{XY|A}. \tag{15}$$

From Equation 15 it is clear that the manifest partial correlation computed on Σ is a biased estimator of the latent partial correlation, and that this bias is due only to the measurement error Ψ. Therefore, under an appropriate correction for attenuation, the manifest partial correlation becomes an unbiased estimate of the latent partial correlation.

It is readily shown that the usual correction for attenuation for the partial correlation (Stouffer, 1936) is also applicable in this situation. This is both surprising and convenient. It is surprising because the correction for the partial correlation is motivated through the true score theory, and in this case X and Y are not compatible with true score theory due their multidimensionality. It is convenient because it means that only the three indicators and their reliabilities are required to obtain an unbiased estimate of the latent partial correlation. If the indicators have known reliability coefficients, the sample size requirements are equivalent to those for estimating the zero-order correlations (e.g., $N \geq 30$). Thus the result is suitable for application in correlation studies. We illustrate the use of the attenuation correction for the partial correlation in the following section.

Summary

This section has shown that the partial correlation between θ_X and an external variable θ_Y conditional on θ_A is invariant under confound indeterminacy. Moreover, this correlation can be estimated using the usual correction for attenuation for the manifest partial correlation. This result allows researchers to control for the confounding effects of test anxiety in correlation studies involving aptitude tests.

EMPIRICAL EXAMPLE

This section describes an example from educational testing. The purpose of the example is to illustrate (a) the consequences of the usual CFA model identification constraints on the effects of interest, (b) the use of tau-equivalent interference effects to identify the model, and (c) estimation of the factor correlation and partial correlation. We emphasize at the outset that we preselected indicator variables to suit this purpose, and therefore the example is only illustrative and does not provide a basis for substantive conclusions.

The data were obtained from voluntary pretesting of 898 high school students registered for the 2009 Vestibular examinations at the University of Brasilia. Prior to sitting for four aptitude tests (humanities, linguistics, math, and natural science), students responded to a questionnaire containing 39 five-point Likert items about cognitive, emotional, and physical aspects of test anxiety. The aptitude tests were administered according to an incomplete block design in which each test consisted of four blocks of 15 items and each student responded to three blocks from each test. The full data set and the test anxiety items are described in detail by da-Silva and Gomes (2011).

Our analyses were necessarily performed at the item level. We employed the weighted least square estimator as implemented in M*plus* 5.21 (Muthén & Muthén, 2009). This estimator is explicitly related to the foregoing discussion of continuous indicators via the polychoric correlation matrix (see, e.g., Muthén, 1978). Our main conclusions were not affected by use of other estimators (e.g., marginal maximum likelihood).

This example made use of $N = 382$ complete responses to nine items from both the linguistics (L) and natural science (NS) examinations. We also made use of items 7, 26, 30, 31, and 34 of the test anxiety questionnaire, which were all related to the experience of distracting thoughts (e.g., "I feel upset because of thoughts that distract me"). Items were preselected based on their unidimensional measurement models and the correlations among their sumscores.

The distraction items showed a stronger sumscore correlation with test performance than the other test anxiety items, which is in keeping with previous research (e.g., Oostdam & Meijer, 2003; Parks-Stamm et al., 2007). However, none of the correlations were very large ($-0.2 < r < 0$). Examination of the data showed that most students reported low test anxiety regardless of their subsequent performance. We interpret this to mean that students perceived the voluntary pretesting context as lowstakes. Nonetheless, the sumscore correlations for the selected items were reasonable from the perspective of the literature on test anxiety (Reeve et al., 2009). The correlations are reported in Table 1.

Identifying the Model

We fitted the full model in Figure 2 with three different identification constraints. The results are summarized in Table 2. The first two models are just identified, and the third is properly nested within the just-identified model. As reported in the second and third columns, the overall fit of the models was marginal but sufficient for the purpose of illustration.

In Model 1 the deficit effects for the L and NS factors were set to zero. Table 2 reports the strongest interference effects for two sets of items. Both effects are negative and

TABLE 1
Sumscore Correlations for Linguistics, Natural Sciences, and Test Anxiety

	L	*NS*
NS	.406	
ANX	−.146	−.194

Note. L = linguistics; NS = natural science; ANX = test anxiety. All correlations significant at the .01 level.

TABLE 2
Model Summary for Three Different Identification Constraints

			Interference		*Deficit*		*Factor Correlation*	
Constraints	*RMSEA*	*CFI*	*L*	*NS*	*L*	*NS*	*0-Order*	*Partial*
Model 1	0.066	.928	−.284*	−.307*	—	—	.701*	.701*
Model 2	0.066	.928	.326*	.305*	−.577*	−.523*	.790*	.701*
Model 3	0.064	.929	−.188*	−.361*	−.118	.102	.669*	.689*

Note. RMSEA = root mean square error of approximation; CFI = comparative fit index; L = language items; NS = natural science items. The factor correlation is between L and NS latent variables, with the test anxiety factor held constant for the partial correlation. The reported interference effects are the strongest for each model. The models are described in the text.

*Statistically significant at the .01 level.

significant at the .01 level, which is in keeping with the interference hypothesis. The zero-order and partial correlations between the L and NS factors are equal, which follows from the identification constraints.

In Model 2 we fixed two of the interference effects to zero instead. In particular, we fixed the L and NS items for which the ratio of the interference effect to the ability loading was largest in Model 1. As described in Figure 3, this should lead to a solution that is numerically disparate from Model 1. This conclusion is borne out in Table 2, which shows a sizable deficit effect for both L and NS items, but the largest "interference effects" are in the wrong direction (i.e., more test anxiety leads to better performance on these items). Also note that although the zero-order factor correlation is larger in Model 2 than Model 1, the partial factor correlation is constant. This illustrates the result proven in Equation 14 earlier.

Model 3 was obtained by forcing subsets of the inference effects to be tau-equivalent. We did not have any substantive hypotheses about the interference effects, so the model is merely exploratory. Setting tau-equivalence over all items, or only within the L and NS items, did not result in acceptable fit. Instead we found four sets of equivalent items. The first set contained all items from both tests that did not show interference. The second contained the remaining L items, the interference effect of which is reported in Table 2. The third set contained five NS items with a moderate interference effect; this was substantially larger than that of the L items. The final set contained a single NS item that showed a very strong interference effect, which is reported in Table 2. The chi-square difference test against the just-identified model showed the constraints to be acceptable, $\chi^2\,(13) = 13.33$, $p = .423$. The fit statistics reported in Table 2 corroborate this conclusion. Neither of the resulting deficit effects were significant at the .01 level, and the factor correlations were similar to those of Model 1.

In summary, comparison of Models 1 and 2 indicates the arbitrariness of the direction, numerical magnitude, and statistical significance of the effects of interest in the just-identified model, for cases where the usual CFA constraints are employed. Model 3 shows that it is possible to identify the model using equalities over the interference effects, and that the acceptability of these equalities can be tested against the data. In this example, the resulting model showed no deficit effects but moderate to large interference effects over some but not all test items. This means that test anxiety was associated with worse test performance on some items, but it was not correlated with ability. The discovery of such items has serious implications for educational practice.

Measuring the Association Between Language and Natural Science

We now consider the relationship between the L and NS items in more detail. Table 3 shows various ways of estimating this relationship. The latent correlations are those from Model 3, which we take as the best available estimates. The sumscore correlations are computed in the usual manner, and the corrected correlations used coefficient alpha to correct for attenuation in the sumscore correlations. Sumscores were computed using only the L and NS items that showed tau equivalence in both their ability loadings and their inference effects in Model 3. This precaution was taken to ensure that the sumscore is an appropriate summary statistic.

TABLE 3
Correlation Coeffcients for Linguistics and Natural Science Items

	Sumscore	*Corrected*	*Latent*
0-order	.270	.701	.669
Partial	.256	.686	.689

Note. Sumscore denotes the usual sumscore correlation between language (L) and natural science (NS) items. Corrected denotes the attenuation corrected sumscore correlations. Latent denotes the correlations for Model 3 and are repeated from Table 2. The partial correlations controlled for test anxiety. The sumscore and corrected correlations were computed using subsets of L and NS items identified as tau-equivalent in Model 3.

Referring to Table 3, the sumscore correlations are seen to severely underestimate the latent correlations. Bias of the sumscore correlations is to be expected in the presence of interference effects (e.g., Wicherts & Zand Scholten, 2010) and in our case attenuation also plays an important role because the reliability of the sumscores was not high ($\alpha < .6$). The discrepancy between the sumscore correlations and the latent correlations illustrates the usefulness of the approach taken in this article. The corrected correlations do better, and in particular the corrected partial correlation is an acceptable approximation of the latent partial correlation. This illustrates the result given in Equation 15.

CONCLUSIONS

In this article we have used CFA to model the role of test anxiety in the measurement of ability and in correlation studies involving aptitude tests. Our main conclusions are as follows. In the context of test anxiety, we are faced with a particular type of rotational indeterminacy that we have referred to as confound indeterminacy. The usual parameter constraints employed for model identification in CFA are not a viable option for eliminating confound indeterminacy, because they render the effects of interest uninterpretable. Fortunately, equality constraints among the cross-loadings of the ability indicators on test anxiety serve to identify the model and these constraints can be statistically tested. If such constraints are not tenable for a given data set, we could nevertheless estimate the partial factor correlation between ability and an external variable, while controlling for test anxiety, because this correlation remains invariant under confound indeterminacy. Moreover, the partial factor correlation can be estimated using the usual correction for attenuation, which is useful for researchers who are committed to using observed correlations.

With regard to test anxiety research, the model presented here allows us to disentangle the contributions of ability and test anxiety to test performance. It also allows for unbiased estimation of the correlation between ability and other variables of interest, and can be used to test various substantive hypotheses. With regard to testing practice, application of the model allows for identification of tests and items that are biased against anxious test takers, and at the same time provides a measure of ability that is not confounded by test anxiety. In other words, the model yields reliable ability estimates even for test takers who experience anxiety. In general, the CFA approach makes available a large number of useful results for applications concerning test anxiety.

The proposed model has also motivated some methodological considerations. Although we have only discussed these within the context of test anxiety, we hope that further research will generalize our results. Specifically, it would be nice to have a rigorous definition of confound indeterminacy for an arbitrary number of factors, as well as the parameter constraints that serve to eliminate it. It would also be nice to know which partial correlations are invariant under this indeterminacy. Additionally, although we have focused on the consequences of confound indeterminacy for parameter estimates, it is also important to describe the consequences for their standard errors. Because confound indeterminacy is arguably quite pervasive in social science applications, these results could have broad consequences.

ACKNOWLEDGMENTS

Peter F. Halpin is now at New York University. Paul De Boeck is now at Ohio State University.

FUNDING

This research was supported by a postdoctoral grant from the National Science and Engineering Council of Canada.

REFERENCES

Ackerman, P., & Heggestad, E. D. (1997). Intelligence, personality, and interests: Evidence for overlapping traits. *Psychological Bulletin*, *121*, 219–245.

Araki, N. (1992). Test anxiety in elementary school and junior high school students in Japan. *Stress and Coping*, *5*, 205–215.

Ball, S. (1995). Anxiety and test performance. In C. Spielberger & P. Vagg (Eds.), *Test anxiety: Theory, assessment, and treatment* (pp. 107–113). London: Taylor & Francis.

Bekker, P. A., Merckens, A., & Wansbeek, T. J. (1989). *Identification, equivalent models, and computer algebra*. San Diego, CA: Academic.

Bertua, C., Anderson, N., & Salgado, J. F. (2005). The predictive validity of cognitive ability tests: A UK meta-analysis. *Journal of Occupational and Organizational Psychology*, *78*, 387–409.

Bollen, K. A. (1989). *Structural equations with latent variables*. New York, NY: Wiley.

Bollen, K. A., & Bauldry, S. (2010). Model identification and computer algebra. *Sociological Methods and Research*, *39*, 127–156.

Browne, M. (2001). An overview of analytic rotation in exploratory factor analysis. *Multivariate Behavioral Research*, *36*, 111–150.

Carroll, J. B. (1993). *Human cognitive abilities: A survey of factor-analytic studies*. Cambridge, UK: Cambridge University Press.

Cizek, G. J., & Burg, S. S. (2006). *Addressing test anxiety in a high-stakes environment: Strategies for classrooms and school*. Thousand Oaks, CA: Corwin.

da-Silva, C. Q., & Gomes, A. E. (2011). Bayesian inference for an item response model for modeling test anxiety. *Computational Statistics and Data Analysis*, *55*, 3165–3182.

Deffenbacher, J. L. (1978). Worry, emotionality, and task-generated interference in test anxiety: An empirical test of attentional theory. *Journal of Educational Psychology*, *70*, 248–254.

di Maria, F., & di Nuovo, S. (1990). Gender differences in social and test anxiety. *Individual Differences*, *11*, 525–530.

Dobson, P. (2000). An investigation into the relationship between neuroticism, extraversion and cognitive test performance in selection. *International Journal of Selection and Assessment*, *8*, 99–109.

Furham, A., Forde, L., & Cotter, T. (1998). Personality and intelligence. *Personality and Individual Differences*, *24*, 187–192.

Hembree, R. (1988). Correlates, causes, effects, and treatment of test anxiety. *Review of Educational Reseacrh*, *58*, 47–77.

Hunter, J. E., & Hunter, R. F. (1984). Validity and utility of alternate predictors of job performance. *Psychological Bulletin*, *96*, 72–98.

Maplesoft. (2005). *Maple version 10* [Computer software]. Waterloo, Canada: Maplesoft.

McCarthy, J. M., & Goffn, R. D. (2005). Selection test anxiety: Exploring tension and fear of failure across the sexes in simulated selection scenarios. *International Journal of Selection and Assessment*, *13*, 282–295.

McCullagh, P., & Nelder, J. A. (1989). *Generalized linear models*. New York, NY: Chapman & Hall.

McDonald, A. S. (2001). The prevalence and effects of test anxiety in school children. *Educational Psychology*, *21*, 89–102.

Meijer, J., & Oostdam, R. (2007). Test anxiety and intelligence testing: A closer examination of the stage-fright hypothesis and the influence of stressful instruction. *Anxiety, Stress & Coping*, *20*, 77–91.

Millsap, R. (2001). When trivial constraints are not trivial: The choice of uniqueness constraints in confirmatory factor analysis. *Structural Equation Modeling*, *8*, 1–17.

Moutafi, J., Furnham, A., & Tsaousis, I. (2006). Is the relationship between intelligence and trait neuroticism mediated by test anxiety? *Personality and Individual Differences*, *40*, 587–597.

Muthén, B. O. (1978). Contributions to the factor analysis of dichotomous variables. *Psychometrika*, *43*, 551–560.

Muthén, L. K., & Muthén, B. O. (2009). *M plus*. Los Angeles, CA: Muthén & Muthén.

Oostdam, R., & Meijer, J. (2003). Influence of test anxiety on measurement of intelligence. *Psychological Reports*, *92*, 3–20.

Parks-Stamm, E., Gollwitzer, P. M., & Oettingen, G. (2007). Action control by implementation intentions: Effective cue detection and efficient response initiation. *Social Cognition*, *25*, 247–264.

Pekrun, R. (1992). *Expectancy-value theory of anxiety: Overview and implications*. Washington, DC: Hemisphere.

Pintrich, P., & Groot, E. D. (1990). Motivational and self-regulated learning components of classroom academic performance. *Journal of Educational Psychology*, *82*, 33–50.

Proost, K., Derous, E., Schreurs, B., Hagtvet, K. A., & Witte, K. D. (2008). Selection test anxiety: Investigating applicants' self- vs other-referenced anxiety in a real selection setting. *International Journal of Selection and Assessment*, *16*, 14–26.

Putwain, D. W. (2007). Test anxiety in UK schoolchildren: Prevalence and demographic patterns. *Journal of Educational Psychology*, *77*, 579–593.

Putwain, D. W., & Daniels, R. A. (2010). Is the relationship between competence beliefs and test anxiety influenced by goal orientation? *Learning and Individual Differences*, *20*, 8–13.

Reeve, C. L., & Bonaccio, S. (2008). Does test anxiety induce measurement bias in cognitive ability tests. *Intelligence*, *36*, 526–538.

Reeve, C. L., Heggestad, E. D., & Lievens, F. (2009). Modeling the impact of test anxiety and test familiarity on the criterion-related validity of cognitive ability tests. *Intelligence*, *37*, 34–41

Salgado, J. F., & Anderson, N. (2003). Validity generalization of GMA tests across the European countries. *European Journal of Work and Organizational Psychology*, *12*, 1–17.

Satorra, A., & Bentler, P. M. (2001). A scaled difference chi-square test for moment structure analsyis. *Psychometrika*, *66*, 507–514.

Stouffer, S. A. (1936). Evaluating the effect of inadequately measured variables in partial correlation analysis. *Journal of the American Statistical Association*, *31*, 348–360.

Takane, Y., & de Leeuw, J. (1987). On the relationship between item response theory and factor analysis of discretized variables. *Psychometrika*, *52*, 393–408.

Tobias, S. (1992). *The impact of test anxiety on cognition in school learning*. Lisse, The Netherlands: Swets & Zeitlinger.

van der Embse, N., & Hasson, R. (2012). Test anxiety and high stakes test performance between school settings: Implications for educators. *Preventing School Failure*, *56*, 180–187.

Weems, C. F., Scott, B. G., Taylor, L. K., Cannon, M., Romano, D. M., Perry, A. M., et al. (2010). Test anxiety prevention and intervention programs in schools: Program development and rationale. *School Mental Health*, *2*, 62–71.

Wicherts, J., & Zand Scholten, A. (2010). Test anxiety and the validity of cognitive tests: A confirmatory factor analysis perspective and some empirical findings. *Intelligence*, *38*, 169–178.

Wren, D. G., & Benson, J. (2004). Measuring test anxiety in children: Scale development and internal construct validation. *Anxiety, Stress & Coping*, *17*, 227–240.

Yanai, H., & Ichikawa, M. (2007). Factor analysis. In C. R. Rao & S. Sinharay (Eds.), *Handbook of statistics: Psychometrics* (Vol. 26, pp. 257–298). Amsterdam: Elsivier.

Zeidner, M. (1995). Personality trait correlates of intelligence. In D. Saklofske & M. Zeidner (Eds.), *International handbook of personality and intelligence: Perspectives on individual differences* (pp. 299–314). New York, NY: Plenum.

Zeidner, M. (1998). *Test anxiety: The state of the art*. New York, NY: Plenum.

Zeidner, M., & Safir, M. P. (1989). Sex, ethnic, and social differences in test anxiety among Israeli adolescents. *Journal of Genetic Psychology*, *150*, 175–185.

APPENDIX

In this appendix we use Equations 12 and 13 to prove Equation 14. We begin by computing the transformed factor covariance matrix $R = T^{-1}\Phi T^{-1'}$:

$$\begin{bmatrix} \frac{1}{t_X^2}\left(\left(\frac{t_{Ag}}{t_A}\right)^2 - 2\phi_{Ag}\frac{t_{Ag}}{t_A} + 1\right) & - & - \\ \frac{1}{t_X t_Y}\left(\phi_{XY} - \frac{t_{AY}}{t_A}\phi_{Ag} - \frac{t_{Ag}}{t_A}\phi_{AY} + \frac{t_{Ag}t_{AY}}{t_A^2}\right) & \frac{1}{t_Y^2}\left(\left(\frac{t_{AY}}{t_Y}\right)^2 - 2\phi_{AY}\frac{t_{AY}}{t_A} + 1\right) & - \\ \frac{1}{t_X t_A}\left(\phi_{Ag} - \frac{t_{Ag}}{t_A}\right) & \frac{1}{t_Y t_A}\left(\phi_{AY} - \frac{t_{AY}}{t_A}\right) & \frac{1}{t_A^2} \end{bmatrix} \tag{A.1}$$

Writing $S = \mathrm{diag}(R)^{-1/2}$, the transformed factor correlation matrix is given by $\Phi^* = SRS$. It is clear from inspection of Equation A.1 that the zero-order correlation between θ_X and θ_Y is affected by the rotation. However, it is perhaps surprising to find that this is not the case for the partial correlation between θ_X and θ_Y when controlling for θ_A.

The partial correlation between θ_X and θ_Y before rotation is defined as

$$\phi_{XY|A} = \frac{\phi_{XY} - \phi_{Ag}\,\phi_{AY}}{\sqrt{1-(\phi_{Ag})^2}\sqrt{1-(\phi_{AY})^2}} \tag{A.2}$$

The transformed partial correlation, $\phi^*_{XY|A}$, can be obtained by computing its elements directly from R. Letting $\sigma^{*2}_{(\cdot)}$ denote the diagonal elements of R, we have in the numerator of $\phi^*_{XY|A}$:

$$\begin{aligned} \phi^*_{AX}\phi^*_{AY} &= \frac{\frac{1}{t_X t_Y t_A^2}\left(\phi_{AX}\phi_{AY} - \frac{t_{AY}}{t_A}\phi_{AX} - \frac{t_{AX}}{t_A}\phi_{AY} + \frac{t_{AX}t_{AY}}{t_A^2}\right)}{\sigma^*_X\,\sigma^*_Y\,\sigma^{2*}_A} \\ &= \frac{\phi_{AX}\phi_{AY} - \frac{t_{AY}}{t_A}\phi_{AX} - \frac{t_{AX}}{t_A}\phi_{AY} + \frac{t_{AX}t_{AY}}{t_A^2}}{t_X\,t_Y\,\sigma^*_X\,\sigma^*_Y} \end{aligned} \tag{A.3}$$

from which it follows that

$$\phi^*_{XY} - \phi^*_{AX}\phi^*_{AY} = \frac{\phi_{XY} - \phi_{AX}\phi_{AY}}{t_X\,t_Y\,\sigma^*_X\,\sigma^*_Y}. \tag{A.4}$$

For the denominator of $\phi^*_{XY|A}$:

$$1 - \left(\phi^*_{AX}\right)^2 = 1 - \frac{\frac{1}{t_X^2\,t_A^2}\left(\phi_{AX} - \frac{t_{AX}}{t_A}\right)^2}{\sigma^{*2}_X\,\sigma^{*2}_A} = \frac{t_X^2\,\sigma^{*2}_X - \left(\phi_{AX} - \frac{t_{AX}}{t_A}\right)}{t_X^2\,\sigma^{*2}_X} = \frac{1-\phi^2_{AX}}{t_X^2\,\sigma^{*2}_X} \tag{A.5}$$

where the last equality follows directly from expansion of the term in parentheses. A similar argument shows that

$$1 - \left(\phi^*_{AY}\right)^2 = \frac{1-\phi^2_{AY}}{t_Y^2\sigma^{*2}_Y}. \tag{A.6}$$

From inspection of Equations A.4, A.5, and A.6, it is readily seen that t_X, t_Y, σ^*_X, and σ^*_Y cancel out in the numerator and denominator of the transformed partial correlation, leading to Equation 14. Following the same sequence of computations, one finds that both of the other partial correlations among θ_X, θ_Y, and θ_A are affected by rotation; these do not reduce to convenient expressions.