TEACHING MATH TO ELLS: RESOURCES AND STRATEGIES FOR TEACHERS

Richmond Hill H.S.
October 30, 2018

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Agenda

- Seeing the Language and Background Knowledge Demands in Mathematics Talk, Text, Tests
- Frontloading Strategies
  - Language
  - Content
- Lesson Framing Strategies
- Interaction/Practice/Language Use Strategies
- Providing Feedback to Learners: What and How to Provide Language Correction
- Resources for Math Teachers of ELLs
Watch and Listen

- Click here to watch
What do you think you are to do?

Look at the Problem.
Tell Your Partner
What You Think It’s Asking

在美國，美國人使用美元。在委內瑞拉，委內瑞拉人使用玻利瓦。當依蓮娜去年到委內瑞拉拜訪她的祖父母時，1美元相當於1,432玻利瓦。如果依蓮娜買禮物花了42美元，請問她花了多少玻利瓦？
Let’s Go Back and Watch, then Discuss:

Did you learn any math?

How did you capture what to do?
(Did visuals help, Did her body language help? Did watching other students help?)

What were you able to do with the print?

How was her rate of speech?

Did you learn any Chinese to help you with other problems you may see on tests?
In America, Americans use dollars, while in Venezuela, Venezuelans use Bolivar(s). Last year, when Elena went to Venezuela to visit her grandparents, one dollar equaled 1,432 Bolivar(s). Elena spent 42 dollars on buying gift(s). How many (much) Bolivars did she spend?
What affects performance when working in an L2?

--new orthography/writing system to learn to decode
--word knowledge
--rate of speech; redundancy; recycling of terms
--length of the interaction
--scaffolds provided by the teacher
--willingness to keep working despite not understanding most of what is going on
--success with what you are doing
--level of frustration/anxiety
--prior subject matter knowledge
Adapting Teacher Discourse

- Slow down
- Enunciate (without exaggerating)
- Simplify; Be Consistent
- Limit contractions, pronouns, idioms
- Recycle important information (and terms)
Use Gestures
Use Visuals and Graphics
Demonstrate Your Words
Be Dramatic
Enhance Your Words
Provide Clear Directions

- **SAY**
- **WRITE**
- **SHOW**
Other EL Student-Friendly Supports

- Be sure the student knows your name
- Establish routines so students know what to expect
- Face the class when speaking
- Avoid slang and explain idioms
- Write legibly; create clear diagrams/visuals
- Repeat important information
- Allow students to audio record lessons
- Provide plenty of wait time
- Post procedures and schedules
5 Principles for Teaching Content to English Language Learners

All children deserve equal access to content—regardless of language level or ability. To achieve this equity, Pearson has developed an instructional framework incorporating five essential principles. This instructional framework is based on the research of numerous language experts, including Jim Cummins.

1. **Identify and Communicate Content and Language Objectives**
   - State comprehensible content objectives.
   - Simplify language, e.g., voiced, not same form—same meaning.
   - Paraphrase.
   - Repeat.
   - Avoid idiomatic saying.
   - Be aware of homonyms and multiple-meaning words.
   - Clarify with simplified language, cultural, standard.
   - Check for understanding.

2. **Frontload the Lesson**
   - Activate prior knowledge and related content.
   - Build background by presenting key vocabulary or antecedent ideas and concepts.
   - Preview text by presenting visuals, readings, and/or highlighted text.
   - Set a purpose for reading by clarifying, posing other questions at the end of the text.
   - Make connections by seeing related ideas and other aspects of the lesson.

3. **Provide Comprehensible Input**
   - Visual aids and written content accessible by providing support.
   - Visuals, charts, illustrations, animations, multimedia.
   - Graphics, graphs, slides, tables.
   - Organizers, graphic organizers, visuals.
   - Summaries, bold, audio-visual language.
5 Principles of Teaching ELLs

- Identify Language Targets and Content Objectives
- Frontload the Lesson/Building Background Knowledge
- Provide Comprehensible Input
- Enabling Language Production/Extending Language
- Assessment of Language and Content Objectives
Research-based instructional design

5 principles for teaching ELLs

1. Identify and Communicate Content and Language Objectives
2. Frontload the Lesson
3. Provide Comprehensible Input
4. Enable Language Production
5. Assess for Content and Language Understanding

Explicitly stated objectives in each lesson/module

Motivate
ENGAGE
SUCCESS

Differentiate
Assess

Opening activities connect to and assess prior knowledge

Hands-on activities utilize multiple modes of instruction: visual, verbal, aural, kinesthetic

Students express their understanding through speaking and writing activities; reflect on successful strategies

Pair and group work promotes language practice in both listening/speaking and reading/writing activities
Research-based instructional design

5 principles for teaching ELLs

1. Identify and Communicate Content and Language Objectives
2. Frontload the Lesson
3. Provide Comprehensible Input
4. Enable Language Production
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- Explicitly stated objectives in each lesson/module
- Opening activities connect to and assess prior knowledge
- Motivate and ENGAGE
- Differentiate instruction
- Assess

Students express their understanding through speaking and writing activities; reflect on successful strategies

Pair and group work promotes language practice in both listening/speaking and reading/writing activities

Hands-on activities utilize multiple modes of instruction: visual, verbal, aural, kinesthetic
Language Learning Cycle

Comprehensible Input

Interaction and Practice

Feedback and Expansion of Language Repertoire
# Planning Language and Content Objectives

## Language
- Content-specific Vocabulary
- General Academic Vocabulary
- Structures (grammar)

## Content
- Knowledge
  - From NYS Standards; NYC Curriculum
- Skills
“If only they understood the question, they could answer it. They know the content, they just don’t know enough English.”

Seeing the Language and Background Knowledge Demands in Math Talk, Text, Tests
Vocabulary Targets:

1. Content-Specific Vocabulary
2. General Academic Vocabulary
<table>
<thead>
<tr>
<th>Mathematics High School</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolute error</td>
<td>Absolute function</td>
</tr>
<tr>
<td>Absolute value</td>
<td>Combination</td>
</tr>
<tr>
<td>Acceleration</td>
<td>Completing the square</td>
</tr>
<tr>
<td>Add radical expressions</td>
<td>Complex number</td>
</tr>
<tr>
<td>Addition reducing procedure</td>
<td>Complex conjugates</td>
</tr>
<tr>
<td>Algebraic function</td>
<td>Compound-event</td>
</tr>
<tr>
<td>Angle of depression</td>
<td>Compound interest</td>
</tr>
<tr>
<td>Arc</td>
<td>Conditional probability</td>
</tr>
<tr>
<td>Area under curve</td>
<td>Confidence interval</td>
</tr>
<tr>
<td>Asymptote of function</td>
<td>Conic section</td>
</tr>
<tr>
<td>Base e</td>
<td>Conjugate-complex number</td>
</tr>
<tr>
<td>Binary system</td>
<td>Continuity</td>
</tr>
<tr>
<td>Bisect</td>
<td>Continuous probability distribution</td>
</tr>
<tr>
<td>Bivariate data</td>
<td>Control group</td>
</tr>
<tr>
<td>Bivariate data transformation</td>
<td>Correlation</td>
</tr>
<tr>
<td>Bivariate distribution</td>
<td>Cosine</td>
</tr>
<tr>
<td>Cartesian coordinates</td>
<td>Critical path method</td>
</tr>
<tr>
<td>Categorical data</td>
<td>Curve fitting</td>
</tr>
<tr>
<td>Central angle</td>
<td>Curve fitting median method</td>
</tr>
<tr>
<td>Central limit theorem</td>
<td>Decibel</td>
</tr>
<tr>
<td>Chord</td>
<td>Density</td>
</tr>
<tr>
<td>Circle without center</td>
<td>Dependent events</td>
</tr>
<tr>
<td>Circular function</td>
<td>Derivation</td>
</tr>
<tr>
<td>Classes of functions</td>
<td>Dilation of object in a plane</td>
</tr>
</tbody>
</table>
Use a List

Progress Monitor

Give Credit for Vocabulary Learning
Tennessee Academic Vocabulary
A Guide for Tennessee Educators

Algebra I

- Absolute value
- Complement of an event
- Compound
- Conjunction
- Direct and inverse variation
- Disjunction
- Domain & range
- Exponential growth (and decay)
- Interest (simple and compound)
- Irrational numbers
- Joint and conditional probability
- Law of Large Numbers
- Mathematical model
- Measure of spread (range, interquartile range)
- Midpoint: formula
- Outlier
- Parent function
- Pascal’s Triangle
- Polynomial (binomial, trinomial)
- Quadratic: formula
- (including discriminant)
- Quantitative and qualitative data
- Radicand
- Rational expression
- Real number properties
- Real roots (zeros, solutions, x-intercepts)
- Relative frequency
- Sequences (arithmetic, geometric, Fibonacci)
- Simulations
- Subsets of real numbers

Algebra II

- Amplitude
- Asymptote
- Binomial Theorem
- Combination
- Common ratio (geometric sequence)
- Complete the square
- Complex conjugate
- Complex number
- Composition (of functions)
- Conic sections (circles, parabola, ellipse, hyperbola)
- Empirical Rule
- Factorial
- Focus (pl. foci)
- Independent and dependent events
- Inverse of a relation
- Logarithm
- Normal distribution
- Period
- Permutation
- Piece-wise function
- Radian measure
- Rational function
- Regression equation
- Series (arithmetic, geometric, finite, infinite, etc.)
- Sigma
- Standard deviation
- Step function
- Synthetic division
- Transcendental function
- Trigonometric function
- Trigonometric identity

Geometry

- Altitude
- Angle of depression
- Angle of elevation
- Apothem
- Arc
- Bisect (bisector)
- Central angle
- Centroid
- Chord
- Circumcenter
- Circumscribed
- Collinear
- Concurrent lines
- Conditional statement
- (including converse, inverse, contrapositive, & biconditional statement)
- Construction
- Convex & concave polygons
- Coplanar
- Corollary
- Deductive & inductive reasoning
- Euclidean & non-Euclidean geometry
- Geometric mean
- Glide reflection
- Incenter
- Inscribed
- Lateral area
- Locus
- Negation
- Oblique
- Orthocenter
- Points of concurrency in a triangle
- Postulate (axiom)
- Proof (formal, two-column, paragraph, flow, coordinate, indirect, counterexample)
- Scalar

- Secant line
- Sector of a circle
- Skew lines
- Tangent line
- Theorem
- Trigonometric ratios (sine, cosine, tangent)
- Undefined terms of geometry
- Vector (magnitude and direction)

Tennessee Department of Education
Timothy K. Webb, Commissioner
July, 2006
Revised: December, 2007
Revised: July, 2009

Unit circle
Variance
<table>
<thead>
<tr>
<th>Sixth Grade</th>
<th>Seventh Grade</th>
<th>Eighth Grade</th>
<th>Algebra I</th>
<th>Algebra II</th>
<th>Geometry</th>
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</thead>
<tbody>
<tr>
<td>algebraic expression</td>
<td>absolute value</td>
<td>adjacent angles</td>
<td>absolute value function</td>
<td>arithmetic/geometric sequences</td>
<td>altitude</td>
</tr>
<tr>
<td>base number</td>
<td>acute triangle</td>
<td>coefficient</td>
<td>ascending/descending</td>
<td>asymptotes</td>
<td>angle of depression/elevation</td>
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<tr>
<td>circumference</td>
<td>alternate interior/exterior angles</td>
<td>constant</td>
<td>binomial</td>
<td>completing the square</td>
<td>angle relationships (complementary, supplementary, etc., expressed algebraically)</td>
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<tr>
<td>complement</td>
<td>bisector</td>
<td>distance formula: (d=rt)</td>
<td>degree of a polynomial</td>
<td>complex numbers</td>
<td>arc (measurement, length, major, minor)</td>
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<tr>
<td>convert</td>
<td>combinations</td>
<td>domain</td>
<td>difference of squares</td>
<td>composition</td>
<td>central angle</td>
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<tr>
<td>coordinate plane</td>
<td>corresponding angles</td>
<td>formula</td>
<td>elimination method (for solving a system of equations)</td>
<td>conic sections</td>
<td>chord</td>
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<tr>
<td>diameter</td>
<td>discount</td>
<td>hypotenuse</td>
<td>factor a polynomial</td>
<td>conjugate (complex)</td>
<td>conditional statements (inverse, converse, contrapositives)</td>
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<tr>
<td>evaluate</td>
<td>equilateral triangle</td>
<td>lateral area</td>
<td>function notation</td>
<td>correlation</td>
<td>congruence</td>
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<tr>
<td>exponent</td>
<td>experimental probability</td>
<td>legs of a triangle</td>
<td>inequalities</td>
<td>curve of best fit</td>
<td>conjecture</td>
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<td>factorization</td>
<td>exponential notation</td>
<td>linear equation</td>
<td>intercepts ((x &amp; y))</td>
<td>delta</td>
<td>construction (protractor, compass, straightedge)</td>
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<tr>
<td>median</td>
<td>integer</td>
<td>linear inequality</td>
<td>irrational numbers</td>
<td>discriminant</td>
<td>convex/concave</td>
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<td>mode</td>
<td>interest</td>
<td>Pythagorean theorem</td>
<td>line of best fit</td>
<td>functions (exponential, polynomial, logarithmic, etc.)</td>
<td>corresponding parts</td>
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<tr>
<td>non-terminating decimal</td>
<td>isosceles triangle</td>
<td>range of a function</td>
<td>linear/nonlinear functions (exponential, quadratic, absolute value)</td>
<td>imaginary</td>
<td>counterexample</td>
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<tr>
<td>order of operations</td>
<td>obtuse triangle</td>
<td>scatter plot</td>
<td>literal equations</td>
<td>inverse function</td>
<td>deductive reasoning</td>
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<tr>
<td>pi</td>
<td>outcome</td>
<td>scientific notation</td>
<td>logarithm</td>
<td>degree formula</td>
<td></td>
</tr>
<tr>
<td>plane figure</td>
<td>parallelogram</td>
<td>slope/intercept form</td>
<td>monomial</td>
<td>Euclidean/non-Euclidean Geometry</td>
<td></td>
</tr>
<tr>
<td>prime factor</td>
<td>permutations</td>
<td>slope</td>
<td>parent graph (linear, absolute value, quadratic, constant)</td>
<td>minimum/maximum (relative, absolute)</td>
<td>inductive reasoning</td>
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<tr>
<td>quadrilateral</td>
<td>polygon</td>
<td>solids (prisms, cones, cylinders, pyramids)</td>
<td>polynomial</td>
<td>normal distribution curve (Gaussian)</td>
<td>inscribed angles and polygons</td>
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<tr>
<td>radius</td>
<td>positive</td>
<td>standard form (of a linear equation)</td>
<td>quadratic equation</td>
<td>parent function (exponential, polynomial, logarithmic)</td>
<td>interior/exterior angles (of a figure)</td>
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<tr>
<td>reciprocal</td>
<td>proportion</td>
<td>surface area</td>
<td>quadratic formula</td>
<td>radical equation</td>
<td>lateral surface area</td>
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<tr>
<td>sequences (arithmetic, geometric, Fibonacci)</td>
<td>quadrant</td>
<td>term</td>
<td>rational expression</td>
<td>standard deviation</td>
<td>midpoint formula</td>
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<td>simplify</td>
<td>rate</td>
<td></td>
<td>real numbers</td>
<td>synthetic division</td>
<td>polyhedra</td>
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<tr>
<td>square units</td>
<td>ratio</td>
<td></td>
<td>relations</td>
<td>three-dimensional coordinate</td>
<td>proof (formal, paragraph, flow, algebraic)</td>
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<td>substitution</td>
<td>regular polygon</td>
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<td>substitution method (for solving a system of equations)</td>
<td>transformation (algebraic)</td>
<td>Pythagorean theorem – area model</td>
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<tr>
<td>supplement</td>
<td>rhombus</td>
<td></td>
<td>rate of change</td>
<td>sigma</td>
<td>median of a triangle</td>
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<td>right triangle</td>
<td></td>
<td>scale factor</td>
<td>rational expression</td>
<td>standard deviation</td>
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<td>scale factor</td>
<td>sciene triangle</td>
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<td>square root</td>
<td>real numbers</td>
<td>midpoint formula</td>
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<tr>
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<td>transversal</td>
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<td>relations</td>
<td>synthetic division</td>
<td>polyhedra</td>
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<td>trapezoid</td>
<td>unit rate</td>
<td></td>
<td>substitution method (for solving a system of equations)</td>
<td>transformation (reflection, rotation, translation)</td>
<td>Pythagorean theorem – area model</td>
</tr>
<tr>
<td>vertical angle</td>
<td></td>
<td></td>
<td></td>
<td>trinomial</td>
<td>reflexive, symmetric and transitive properties</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>weighted averages</td>
<td>secant line</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>zero of a function</td>
<td>tangent line</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>theorem/postulate/conjecture</td>
<td>total surface area</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>transformation (reflection, rotation, translation)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>trigonometric ratio (sine, cosine, tangent)</td>
</tr>
</tbody>
</table>

Source: [http://sde.state.ok.us/curriculum/BAV.pdf](http://sde.state.ok.us/curriculum/BAV.pdf)

Use the Academic Vocabulary List!(AVL, Gardner)

Choosing **Academic Vocabulary** that appears frequently in academic texts
<table>
<thead>
<tr>
<th>#</th>
<th>Word</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>study</td>
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<tr>
<td>2</td>
<td>develop</td>
<td>128974</td>
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<tr>
<td>3</td>
<td>group</td>
<td>125012</td>
</tr>
<tr>
<td>4</td>
<td>system</td>
<td>116141</td>
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<tr>
<td>5</td>
<td>relate</td>
<td>114267</td>
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<tr>
<td>6</td>
<td>research</td>
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<td>7</td>
<td>social</td>
<td>103635</td>
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<td>8</td>
<td>result</td>
<td>96016</td>
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<tr>
<td>9</td>
<td>use</td>
<td>93271</td>
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<tr>
<td>10</td>
<td>provide</td>
<td>93212</td>
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<tr>
<td>11</td>
<td>however</td>
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<td>12</td>
<td>increase</td>
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<td>13</td>
<td>experience</td>
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<td>level</td>
<td>79201</td>
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<td>process</td>
<td>78679</td>
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<td>culture</td>
<td>77470</td>
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<td>17</td>
<td>history</td>
<td>77164</td>
</tr>
<tr>
<td>18</td>
<td>active</td>
<td>76010</td>
</tr>
</tbody>
</table>

**Academic Vocabulary List**

Link to an Established Academic Vocabulary List!

Teach words in word families; in semantic networks

[http://www.academicvocabulary.info/samples/families.pdf](http://www.academicvocabulary.info/samples/families.pdf)
Lesson 1: Unknown Angle Proofs—Proofs of Known Facts

Classwork

Opening Exercise

A proof of a mathematical statement is a detailed explanation of how that statement follows logically from other statements already accepted as true.

A theorem is a mathematical statement with a proof.

Discussion

Once a theorem has been proved, it can be added to our list of known facts and used in proofs of other theorems. For example, in Lesson 9, we proved that \( \text{vertical angles are equal in measure} \), and we know (from earlier grades and by paper cutting and folding) that \( \text{if a transversal intersects two parallel lines, alternate interior angles are equal in measure} \). How do these facts help us prove that corresponding angles are equal in measure?

In the diagram to the right, if you are given that \( \ell \parallel \ell' \), how can you use your knowledge of how vertical angles and alternate interior angles are equal in measure to prove that \( \angle 1 = \angle 2 \)?

You now have available the following facts:

- Vertical angles are equal in measure.
- Alternate interior angles are equal in measure.
- Corresponding angles are equal in measure.
Use any of the following facts to prove the interior angles on the same side of the transversal are supplementary. Add any necessary labels to the diagram below, and then write out a proof including a statement of what needs to be proved.

**Given:** \( \angle 1 \) and \( \angle 2 \) are supplementary.

**Prove:** \( \angle 1 + \angle 2 = 180°. \)

Let's review the theorems we have now proven:
- Vertical angles are equal in measure.
- A transversal intersects a pair of lines. The pair of lines is parallel if and only if:
  - Alternate interior angles are equal in measure.
  - Corresponding angles are equal in measure.
- Interior angles on the same side of the transversal add to 180°. The sum of the degree measures of the angles of a triangle is 180°.

**SideNote:** Take a moment to take a look at one of those really famous Greek guys we hear so much about in geometry, Eratosthenes. Over 2,000 years ago, Eratosthenes used the geometry we have just been working with to find the circumference of Earth. He did not have cell towers, satellites, or any other advanced instruments available to scientists today. The only things Eratosthenes used were his eyes, his feet, and perhaps the ancient Greek equivalent to a protractor.

Watch this video to see how he did it, and try to spot the geometry we have been using throughout this lesson.

https://youtu.be/1er8E3aV44g

Now that you have proven this, you may add this theorem to your available facts.

Interior angles on the same side of the transversal that intersect parallel lines sum to 180°.

Use any of the above facts to prove the three interior angles of a triangle sum to 180°. For this proof, you will need to draw an auxiliary line, parallel to one of the triangle's sides and passing through the vertex opposite that side. Add any necessary labels, and write out your proof.

**Example:**

Construct a proof designed to demonstrate the following:

If two lines are perpendicular to the same line, they are parallel to each other.

(a) Draw and label a diagram, (b) state the given facts and the conclusion to be proved, and (c) write out a clear statement of your reasoning to justify each step.
**Discussion**

Each of the three parallel line theorems has a converse (or reversing) theorem as follows:

<table>
<thead>
<tr>
<th>Original</th>
<th>Converse</th>
</tr>
</thead>
<tbody>
<tr>
<td>If two parallel lines are cut by a transversal, then alternate interior angles are equal in measure.</td>
<td>If two lines are cut by a transversal such that alternate interior angles are equal in measure, then the lines are parallel.</td>
</tr>
<tr>
<td>If two parallel lines are cut by a transversal, then corresponding angles are equal in measure.</td>
<td>If two lines are cut by a transversal such that corresponding angles are equal in measure, then the lines are parallel.</td>
</tr>
<tr>
<td>If two parallel lines are cut by a transversal, then interior angles on the same side of the transversal are supplementary.</td>
<td>If two lines are cut by a transversal such that interior angles on the same side of the transversal are supplementary, then the lines are parallel.</td>
</tr>
</tbody>
</table>

Notice the similarities between the statements in the first column and those in the second. Think about when you would need to use the statements in the second column, that is, the time when you are trying to prove what lines are parallel.

**Example 2**

In the figure to the right, \( z_1 = 1 \). Prove that \( z_3 = 1 \).

**Problem Set**

1. Given \( \angle 1 \) and \( \angle 2 \) are supplementary, and \( \angle 1 = 1 \). Prove \( \angle 2 = 1 \).

2. An other theorem states that \( \overline{AB} \parallel \overline{CD} \) if and only if \( \angle 1 \) and \( \angle 2 \) are supplementary. Prove this theorem. If \( \overline{AB} \parallel \overline{CD} \), state the given information and the theorem to be proven, and list the necessary steps to demonstrate the proof.
Choose some vocabulary targets (8-10 terms to really give a workout):

<table>
<thead>
<tr>
<th>Content Specific Vocabulary</th>
<th>AVL Words; Cross-Disciplinary Academic Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proof</td>
<td>State (Statement) (^1)53</td>
</tr>
<tr>
<td>Theorem</td>
<td>Diagram(^6)3)8</td>
</tr>
<tr>
<td>Prove/Proof</td>
<td>Supplement (supplementary)(^7)51</td>
</tr>
<tr>
<td>Vertical</td>
<td>Intersect (^1)3)2)5</td>
</tr>
<tr>
<td>Angle</td>
<td>Auxiliary(^1)4)5)7</td>
</tr>
<tr>
<td>Equal measure</td>
<td>Parallel(^5)6)5</td>
</tr>
<tr>
<td>Alternate interior angles</td>
<td>Conjecture(^1)4)0)8</td>
</tr>
<tr>
<td>Corresponding Angles</td>
<td>Justify (^3)9)4</td>
</tr>
<tr>
<td>Transversal</td>
<td>Converse(^9)1)2</td>
</tr>
<tr>
<td>Triangale</td>
<td>Construct(^2)6)8</td>
</tr>
<tr>
<td>Circumference</td>
<td>Demonstrate(^1)9)9</td>
</tr>
</tbody>
</table>

\(^x + y \text{ sum to}....(\text{sum as verb})\)

\(\text{Diagram}\) \(\text{Intersect}\) \(\text{Auxiliary}\) \(\text{Parallel}\) \(\text{Conjecture}\) \(\text{Justify}\) \(\text{Converse}\) \(\text{Construct}\) \(\text{Demonstrate}\)
Teach 8-10 vocabulary words well in every “unit”

- Choose key AVL words that come up a lot in the unit
- Select 2-3 of these for each unit
- Make sure that you teach these words as part of a word family; teach at least 3 words from the family so kids get experience with different forms of the same word
- Make sure to give practice to these words as much as the content words

Key Content Words

- Do not teach words students will understand from reading the text (words that are clear from the context; words that are glossed in the text)
- If the word has a cognate students know, skip it
- Make sure to choose different types of words: not just nouns but verbs, adjectives, adverbs
- Not just words but also “lexical bundles”
Forms of a word; Word endings (-al; -ment, -tion, -ing, -ary)

- Mathematical
- Transversal
- Statement
- Explanation
- Information
- Corresponding
- Passing
- Supplementary
- Necessary
<table>
<thead>
<tr>
<th>Prefix</th>
<th>Word</th>
<th>Suffix</th>
<th>New Word</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mathematics</td>
<td>-al</td>
<td>mathematical</td>
<td></td>
</tr>
<tr>
<td></td>
<td>state</td>
<td>-ment</td>
<td>statement</td>
<td></td>
</tr>
</tbody>
</table>
### Responding to Writing Prompts

To write an effective response to a writing prompt, you need to determine more than just your topic, audience, and form. You also need to be able to recognize the key words or directions in the prompt and to know how to respond to those directions appropriately. Familiarizing yourself with the information in the following chart can help you do both of these things.

<table>
<thead>
<tr>
<th>Direction</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>Describe, Identify, Define</td>
<td>Identify the main equalities or distinguishing characteristics of your subject, using specific facts or sensory details.</td>
</tr>
<tr>
<td>Recount, Narrate, Relate, Tell About</td>
<td>Present the sequence of events in a story or the steps in a process in chronological order. Use vivid verbs and sensory details.</td>
</tr>
<tr>
<td>Discuss</td>
<td>Identify the key points or relationships, backing these up with examples, quotations, comparisons, and other details.</td>
</tr>
<tr>
<td>Explain</td>
<td>Present the main points of or important steps in whatever you have been asked to explain, using facts, examples, and reasons to clarify what you mean.</td>
</tr>
<tr>
<td>Compare, Contrast</td>
<td>Show the ways in which two or more things are alike and the ways in which they differ, using examples and other details to support similarity and difference you point out.</td>
</tr>
<tr>
<td>Analyze, Evaluate, Review</td>
<td>Examine the main qualities of your subject to arrive at a conclusion about some aspect of it, such as how well the parts function together as a whole.</td>
</tr>
<tr>
<td>Show Causes and Effects</td>
<td>Present the reasons for and the results of a particular event or situation, using specific details to clarify precisely what happened.</td>
</tr>
<tr>
<td>Persuade, Convince, Express Your Opinion</td>
<td>State your point of view and support it with facts, statistics, examples, quotations, and other sound evidence.</td>
</tr>
<tr>
<td>Interpret</td>
<td>Explain in your own words the meaning of whatever you’ve been asked to interpret, supporting your ideas with facts and other details.</td>
</tr>
<tr>
<td>Summarize</td>
<td>Present a condensed version of a story or a process by relating only the main events or steps and showing how one leads to the next. Do not include supporting details or other types of elaboration.</td>
</tr>
</tbody>
</table>
## THE TWELVE WORDS

Educators have identified 12 words that sometimes trip up students when they are responding to questions in writing or speaking and when taking a test. Understanding and using these 12 words are critical for student success from upper elementary to post-secondary studies. Schools are encouraged to develop school-wide efforts to assure that students become familiar with these key words.

<table>
<thead>
<tr>
<th>DESCRIBE</th>
<th>EXPLAIN</th>
<th>TRACE</th>
<th>SUPPORT</th>
<th>PREDICT</th>
<th>COMPARE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Describe means to:</td>
<td>When you explain something think about a teacher who is teaching you how to do something. They want to describe the process so that you know what to do first, next, and last. When you are asked to explain something you are being asked to share something with enough clarity and detail so that the recipient easily understands it.</td>
<td>Trace means to:</td>
<td>Support means to:</td>
<td>To predict is to:</td>
<td>To compare means to:</td>
</tr>
<tr>
<td>tell about something</td>
<td>say</td>
<td>outline</td>
<td>give facts for something</td>
<td>fortell</td>
<td>think about what things have in common</td>
</tr>
<tr>
<td>show</td>
<td>sketch or draw</td>
<td>sketch or draw</td>
<td>explain why</td>
<td>forecast</td>
<td>check for likenesses, similarities</td>
</tr>
<tr>
<td>illustrate</td>
<td>map out</td>
<td>map out</td>
<td>prove it</td>
<td>give someone reasons</td>
<td>match up things</td>
</tr>
<tr>
<td>list the attributes of a thing</td>
<td>copy</td>
<td>follow from the beginning</td>
<td>use examples from something</td>
<td>back up your point of view</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CONTRAST</th>
<th>SUMMARIZE</th>
<th>INFER</th>
<th>ANALYZE</th>
<th>EVALUATE</th>
<th>FORMULATE</th>
</tr>
</thead>
<tbody>
<tr>
<td>To contrast means to:</td>
<td>Summarize means to:</td>
<td>Infer means to:</td>
<td>Analyze means to:</td>
<td>Evaluate means to:</td>
<td>Formulate means to:</td>
</tr>
<tr>
<td>identify how things are unlike, or opposite</td>
<td>recall</td>
<td>suggest</td>
<td>take it apart</td>
<td>grade</td>
<td>build</td>
</tr>
<tr>
<td>check for differences</td>
<td>shorten</td>
<td>conclude</td>
<td>break it down</td>
<td>rank</td>
<td>add up</td>
</tr>
<tr>
<td></td>
<td>a brief description</td>
<td>fill in the blanks</td>
<td>examine</td>
<td>rate</td>
<td>plan</td>
</tr>
<tr>
<td></td>
<td>list the main points</td>
<td>decide the meaning of</td>
<td>explore</td>
<td>review a performance</td>
<td>decide what is good or bad about something</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>investigate</td>
<td>judge the quality</td>
<td>make it</td>
</tr>
</tbody>
</table>

When you contrast things ask yourself “How are they different?” Apples and oranges are fruits but there are many differences. When you are contrasting things in your writing or speaking you are describing how they are different from each other.

Do you see the word “sum” in summarize? Think about adding or summing up a thing. When you are asked to summarize, you create a short piece about something by putting it in your own words.

When you hear the word infer ask yourself, “What does this make me think?” What is the author trying to tell me without using the exact words?” Infer can mean to read between the lines. In writing or speaking when you are asked to infer, it means that you think about what can you suggest or conclude from what you have read?

When you analyze something you look at it closely. When you think of analyze think of someone taking a bicycle apart to explore its parts and take a closer look at it. When you analyze something you are breaking it down to take a closer look at it before you speak or write about it.

Think about the athletes at the Olympics. The judges evaluate what is good or bad about their performance and give them a ranking or grade. When you are asked to evaluate something you think about what is good or bad about that particular thing and then write or speak about it.

When you formulate something, you put it all together in a plan with details. When you hear the word formulate, think about putting a puzzle together. When you are asked to formulate in your writing or speaking you will create a piece that constructs or pulls things together into a plan.

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2 We believe that the source of the Twelve Words is Larry Bell, educational consultant.

3 To view a simple slide show on the twelve words above, go to: http://www.owensboro.k12.ky.us/edtech/12words/12words.htm
Prepositions:

e.g. *In* the diagram *to* the right....

Find four solutions *to* the equation \((X^2 - 9) (x^3 - 8) = 0\)

How vertical angles and alternate interior angles are equal *in* measure *to* prove....

What is the degree *of* your polynomial?

Can you find a rule that relates the multiplicities of the zeros *to* the degree *of* the polynomial function?
Lexical bundles/Phrases/Transitions:

<table>
<thead>
<tr>
<th>Purpose</th>
<th>Transition Words</th>
</tr>
</thead>
<tbody>
<tr>
<td>Giving a definition</td>
<td>is equal to means refers to is synonymous with</td>
</tr>
<tr>
<td>Providing an example</td>
<td>is the same as in other words consists of in fact</td>
</tr>
<tr>
<td>Including to illustrate</td>
<td></td>
</tr>
<tr>
<td>Suggesting more ideas</td>
<td>furthermore also finally another moreover</td>
</tr>
<tr>
<td>Sequencing</td>
<td>first...second next initially before</td>
</tr>
<tr>
<td></td>
<td>preceding when finally after</td>
</tr>
<tr>
<td></td>
<td>following as not long after now</td>
</tr>
<tr>
<td>Comparing</td>
<td>same as just like/as in the same way in comparison</td>
</tr>
<tr>
<td></td>
<td>not only...but also as well as similarly</td>
</tr>
<tr>
<td>Contrasting</td>
<td>different from as opposed to instead of in contrast</td>
</tr>
<tr>
<td></td>
<td>however but although yet</td>
</tr>
<tr>
<td></td>
<td>while on the other hand</td>
</tr>
<tr>
<td>Showing cause and effect relationships</td>
<td>because as a result of may be due to since</td>
</tr>
<tr>
<td></td>
<td>consequently this led to so that nevertheless</td>
</tr>
<tr>
<td></td>
<td>in order to effects of for this reason if ... then</td>
</tr>
<tr>
<td></td>
<td>therefore thus</td>
</tr>
<tr>
<td>Describing problems and solutions</td>
<td>one answer is one reason is a solution is the problem is</td>
</tr>
<tr>
<td>Expressing an opinion or conclusion</td>
<td>I think I believe that I predict that I suggest that</td>
</tr>
<tr>
<td></td>
<td>I conclude that I deduce that I speculate that in my opinion</td>
</tr>
<tr>
<td></td>
<td>I agree with that</td>
</tr>
<tr>
<td>Reporting findings or outcomes</td>
<td>I/We found that I/We learned that I/We discovered that I/We observed that</td>
</tr>
</tbody>
</table>

Source: Adapted with permission from English Learners and the Language Arts (ELLA). (2003). San Francisco WestEd.
Components of Academic Language

- Bricks: vocabulary
- Mortar: Grammar/syntax/form
- Foundation: Language functions

Zwiers, 2008
The Demands of Mathematics

**Write** two functions using the expressions on each side of the equation $6+1 = (x +7)^2$. **Graph** the functions.

**Add** whole numbers and add fractions. **Compare** with the estimate. **You** can also find the sum using a model.

In the shopping mall giveaway, each store represents an outcome. If you want to pick a food store, the 25 food stores are favorable outcomes. If each outcome has an equal chance of happening, you can find theoretical probability using this formula.

**Lexical bundles:** you can use a formula to find the area of a square or the volume of a cube.
Consider the polynomial function $Qx=x^6-3x^5+4x^3-12x^2+x-3$.

a) Divide $Q$ by the divisor $(x-3)$ and rewrite in the form $Q(x)=(\text{divisor})(\text{quotient}) + \text{remainder}$.

b) Evaluate $Q(3)$.

In the diagram to the right, if you are given that $AB \parallel CD$, how can you use your knowledge of how vertical angles and alternate interior angles are equal in measure to prove that $x=w$?
Mathematics Discourse Features

- conceptually packed.
- high-density of important words.
- require up-and-down as well as left-to-right eye movements.
- require reading-rate adjustment.
- require multiple readings.
- use numerous symbolic devices.
- contain a great deal of technical language with precise meaning.
### Cross-Cultural Differences in Notation and Operations

#### Figure 4.2: Mathematical Notational and Procedural Comparisons Between U.S. and Latin American Countries

<table>
<thead>
<tr>
<th></th>
<th>United States</th>
<th>Latin American Countries</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Reading Numbers</strong></td>
<td>Numbers with 10, 11, or 12 digits are designated as billions. 8,000,000,000 is read &quot;eight billion&quot;</td>
<td>Numbers with at least 13 digits are designated as billions. 8,000,000,000 is read &quot;eight thousand million&quot; 8,000,000,000,000 is read &quot;eight billion&quot;</td>
</tr>
<tr>
<td><strong>Separation of digits in large numbers</strong></td>
<td>Comma</td>
<td>Decimal point, space, comma, apostrophe, or semicolon</td>
</tr>
<tr>
<td><strong>Negative Numbers</strong></td>
<td>Preceding negative sign (-)</td>
<td>Preceding negative sign (-) or bar over the number</td>
</tr>
<tr>
<td><strong>Repeating Decimals</strong></td>
<td>Bar over repeating digits or ellipsis following digits 0.33 = 0.333...</td>
<td>Arc over repeating digits 0.32</td>
</tr>
<tr>
<td><strong>Decimal Fractions</strong></td>
<td>Decimal point 4.56</td>
<td>Comma 4.56</td>
</tr>
<tr>
<td><strong>Operation Symbols</strong></td>
<td>Colon denotes division primarily in ratios.</td>
<td>Colon is one of four symbols denoting division.</td>
</tr>
<tr>
<td><strong>Angle Notation</strong></td>
<td>Angle symbol to the left of angle name</td>
<td>Angle symbol above angle name</td>
</tr>
<tr>
<td><strong>Measurement</strong></td>
<td>US Customary</td>
<td>Metric</td>
</tr>
<tr>
<td><strong>Prime Factorization</strong></td>
<td>Factor Tree</td>
<td>Vertical Line</td>
</tr>
<tr>
<td><strong>Division of Fractions</strong></td>
<td>Invert second fraction and then multiply</td>
<td>Cross-multiply</td>
</tr>
<tr>
<td><strong>Least Common Multiple</strong></td>
<td>Use prime factorization</td>
<td>Multiply common prime factors and the prime factors that appear in each number</td>
</tr>
<tr>
<td><strong>Subtraction Algorithm</strong></td>
<td>Renaming method 42 - 19 = (30 + 12) - (10 + 9)</td>
<td>&quot;Equal additions&quot; method 42 - 19 = (40 + 2) - (10 + 9) = (40 + 12) - (20 + 9)</td>
</tr>
<tr>
<td><strong>Division Algorithm</strong></td>
<td>&quot;Long&quot; division</td>
<td>Rely more on mental mathematics or &quot;short&quot; division</td>
</tr>
<tr>
<td><strong>Parentheses</strong></td>
<td>Evaluate within parentheses first</td>
<td>Use distributive property</td>
</tr>
<tr>
<td><strong>Algebraic Equations</strong></td>
<td>Perform operations on both expressions of the equation x + 35 = 75 x = 40</td>
<td>Mentally find missing number in each operation x + 35 = 75 What plus 35 equals 75? 40 + 35 = 75</td>
</tr>
</tbody>
</table>
# Academic Language of Mathematics

## TEXT/TALK FEATURES
- conceptually packed
- high density of unique words with specific meanings
- great deal of technical language with precise meanings
- requires multiple readings
- requires a reading rate adjustment because text must be read more slowly than natural language texts
- uses numerous symbols
- many charts and graphs

## MAJOR TEXT STRUCTURES AND FEATURES OF TALK
- cause and effect; comparisons; logical or chronological sequence

## SUBJECT MATTER–SPECIFIC VOCABULARY
- e.g., divisor, denominator, integer, quotient, coefficient, equation, protractor, place value, proper/improper fraction

## WORDS USED IN NEW WAYS
- e.g., table, column, variable, carry, irrational/rational, mean, factor, term, expression, odd, set

## MULTIPLE WAYS OF SAYING THE SAME THING (SYNONYMS)
- e.g., add, plus, combine, and, sum, increased by, total; subtract from, decreased by, less, minus, differ, less than, have left

## COGNATES (SPANISH/ENGLISH)
- e.g., base/base; centimetro/centimeter; columna/column; concepto/concept; número/number; ordinal/ordinal; grupo/group; identificar/identify; secuencia/sequence; angulo/angle; circulo/circle; diferencia/difference; dividir/divide lineal/line; multiplicar/multiply

## PHRASES WITH SPECIFIC MEANINGS; LEXICAL BUNDLES
- e.g., least common multiple, standard deviation, square root, a quarter of, divided by vs. divided into, as much as, common factor, the size of the, greater than or equal to, not more than

## TRANSITION WORDS; LOGICAL CONNECTORS
- if . . . then, if and only if, because, that is, for example, such that, but, consequently, either

## COMMON COMMUNICATIVE FUNCTIONS
- following directions in a sequence, show, tell, ask and answer factual questions, predict, explain, justify, hypothesize, conjecture

## HELPFUL READING/Writing SKILLS AND STRATEGIES
- adjust reading rate, reread difficult text, confirmation checks/summarize as you go, take notes while reading, use graphs, number lines, and charts to complement the understanding of text

For Your Reference
Planning Your Language Objectives

**Language Form**
- Sounds of Language
- Form of words (nouns, pronouns, verbs, adjectives, adverbs)
- Structure of sentences; sentence patterns
- Length, Complexity of Sentences
- Connectors; Cohesive ties

**Language Functions**
- Kinds of communicative tasks students must accomplish (name, tell, compare, describe, explain, apologize, insult, request)

**Language Content & Use**
- Semantics (vocabulary; phrasing; tone)
- Pragmatics (intended meanings; language as used in different contexts)

**Language Style**
- Formal vs. informal/casual
- Social vs. academic
- Colloquial Language (idioms; common similes, metaphors)
Suggested Language Objectives:

**Language Forms**
- Sounds of Language (*th* in theorem, *v* in vertex)
- Parts of Speech (prepositions, commands, comparative/superlative adjectives)
- Sentence Patterns (*clauses*, *modals* (*could*, *may*), possibility—*if...then*, *given*...)
- Length, Complexity of Sentences

**Language Functions**
- Kinds of communicative tasks students must accomplish:
  - Draw and label,
  - find,
  - consider,
  - compare,
  - determine,
  - explain,
  - state, etc.
Common communication in mathematics classrooms

- discussing ideas and asking questions,
- summarizing instructional key ideas or defining a term,
- following and giving instructions,
- recounting how a problem was solved,
- explaining thinking/reasoning aloud,
- giving reasons for a response,
- showing problem solving steps to display knowledge on tests,
- describing procedures to follow.

Make Sentence Frames for These Functions
Frontloading Strategies: Teaching Background Knowledge and Key Vocabulary
1) Joy wants to buy strawberries and raspberries to bring to a party. Strawberries cost $1.60 per pound and raspberries cost $1.75 per pound. If she only has $10 to spend on berries, which inequality represents the situation where she buys $x$ pounds of strawberries and $y$ pounds of raspberries?

(1) $1.60x + 1.75y \leq 10$
(3) $1.75x + 1.60y \leq 10$
(2) $1.60x + 1.75y \geq 10$
(4) $1.75x + 1.60y \geq 10$

2) On the main floor of the Kodak Hall at the Eastman Theater, the number of seats per row increases at a constant rate. Steven counts 31 seats in row 3 and 37 seats in row 6. How many seats are there in row 20?

(1) 65  
(3) 69  
(2) 67  
(4) 71
3) Which situation is not a linear function?
(1) A gym charges a membership fee of $10.00 down and $10.00 per month.
(2) A cab company charges $2.50 initially and $3.00 per mile.
(3) A restaurant employee earns $12.50 per hour.
(4) A $12,000 car depreciates 15% per year.

4) The amount Mike gets paid weekly can be represented by the expression
2.50a + 290, where a is the number of cell phone accessories he sells that week. What is the constant term in this expression and what does it represent?
(1) 2.50a, the amount he is guaranteed to be paid each week
(2) 2.50a, the amount he earns when he sells a accessories
(3) 290, the amount he is guaranteed to be paid each week
(4) 290, the amount he earns when he sells a accessories

5) There are two parking garages in Beacon Falls. Garage A charges $7.00 to park for the first 2 hours, and each additional hour costs $3.00. Garage B charges $3.25 per hour to park.

When a person parks for at least 2 hours, write equations to model the cost of parking for a total of x hours in Garage A and Garage B.
Things ELLs may not have experience with:

- American names (Joy, Steven, Mike, Dylan)
- Foods common in US (strawberries, raspberries)
- Activities common in US (parties, gym memberships, cabs, parking garages; buying & selling cell phone accessories, campgrounds)
- Measurement/Monetary systems in US (pounds, miles, currency and coins)
- Places (Kodak Hall at the Eastman Theater)
- How public spaces are organized (seats in rows)
- Ways we value merchandise (automobile depreciation).
How to Help ELLs over cultural and linguistic hurdles:

- Make sure they know about pounds (lbs.), dimes, quarters, feet, cubic feet, miles
- Help recognize names of people (capitalized in sentences doesn’t matter if male or female (but they may get a clue in the sentence if it says “he”/”she”)
- Teach symbols like $, % ≥ ≤ and the words for these (e.g. dollars, percent...) as items may contain symbols or words
- Make sure they capture/look for negation (is NOT)
- Teach lexical bundles like “at least”, “for a total of”, “If... only”, “must be true”, “to the nearest tenth”, “approximate value”
More things you can help with:

- Capturing the meaning of prepositions common in math: inside, of, in, into, for, on, at, per
- Practice deciphering the meaning of “if” clauses
- Help with infinitives: to be paid (e.g. (wants to be paid, to bring, to spend
- Help with the command words in problems: Justify, state, explain, determine, display, represent, show, model, write…
- Make sure they capture comparatives and superlatives (greater, greatest)
- Help them capture verbs of possibility (modals): must be, can be
Offer Templates Students Can Use
Connect to and Assess Prior Knowledge—Ask:

- What do you see in the lesson that might be unfamiliar to your ELLs?

- What prior knowledge is needed to understand this lesson? (concepts upon which the lesson depends?)
Survey Student Prior Learning

Do quick assessments (quick writes, exit slips in L1 or L2)

Review student records to see what math courses they have had
http://www.edugains.ca/newsite/ell/index.html
In order to choose 8-10 terms to really give a workout (see next slide):

**Content Specific Vocabulary**
- Proof
- Theorem
- Prove/Proof
- Vertical
- Angle
- Equal measure
- Alternate interior angles
- Corresponding Angles
- Transversal
- Triangle
- \((x + y \text{ sum to})\)
- \((x + y \text{ sum as verb})\)
- Vertex
- Draw and Label
- Write out (write out your proof)
- Circumference
- Protractor
- Perpendicular
- 

**AVL Words; Cross-Disciplinary Academic Terms**
- State (Statement) 153
- Diagram 638
- Supplement (supplementary) 751
- Intersect 1325
- Auxiliary 1457
- Parallel 565
- Conjecture 1408
- Justify 394
- Converse 912
- Construct 268
- Demonstrate 199
Find Out What Key Terms Students Know:

**Vocabulary in Context**

<table>
<thead>
<tr>
<th>word</th>
<th>I recognize it in context, I think it has something to do with</th>
<th>I have never seen the word before, so to learn about it I will</th>
<th>I have heard of the word, but I don't know what it means. To understand it, I am going to</th>
<th>I know the word, it means</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
<td>Use L1 or L2</td>
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</tbody>
</table>

Place “candidate” words in first row

Have students complete before you start the unit
Relevant Vocabulary Terms

In the definitions below, the symbol $R$ stands for the set of real numbers.

**FUNCTION:** A function is a correspondence between two sets, $X$ and $Y$, in which each element of $X$ is assigned to one and only one element of $Y$.

The set $X$ in the definition above is called the **domain** of the function. The **range** (or **image**) of the function is the subset of $Y$, denoted $f(X)$, that satisfies the following property: $y$ is an element of $f(X)$ if and only if there is an $x$ in $X$ such that $f(x) = y$.

If $f(x) = x^2$ where $x$ can be any real number, then the domain is all real numbers (denoted $R$), and the range is the set of nonnegative real numbers.

**POLYNOMIAL FUNCTION:** Given a polynomial expression in one variable, a polynomial function in one variable is a function $f: R \to R$ such that for each real number $x$ in the domain, $f(x)$ is the value found by substituting the number $x$ into all instances of the variable symbol in the polynomial expression and evaluating.

It can be shown that if a function $f: R \to R$ is a polynomial function, then there is some non-negative integer $n$ and collection of real numbers $a_0, a_1, a_2, \ldots, a_n$ with $a_n \neq 0$ such that the function satisfies the equation $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$, for every real number $x$ in the domain. The function $f(x) = 3x^3 + 4x^2 + 4x + 7$, where $x$ can be any real number, is an example of a polynomial function written in standard form.
**Degree of a Polynomial Function:** The *degree of a polynomial function* is the degree of the polynomial expression used to define the polynomial function. The degree of $f(x)=8x^3+4x^2+7x+6$ is 3, but the degree of $g(x)=x+12-x-12$ is 1 because when $g$ is put into standard form, it is $g(x)=4x$.

**Constant Function:** A constant function is a polynomial function of degree 0. A constant function is of the form $f(x)=c$, for a constant $c$.

**Linear Function:** A linear function is a polynomial function of degree 1. A linear function is of the form $f(x)=ax+b$, for constants $a$ and $b$ with $a\neq 0$.

**Quadratic Function:** A quadratic function is a polynomial function of degree 2. A quadratic function is in *standard form* if it is written in the form $f(x)=ax^2+bx+c$, for constants $a$, $b$, $c$ with $a\neq 0$ and any real number $x$.

**Cubic Function:** A cubic function is a polynomial function of degree 3. A cubic function is of the form $f(x)=ax^3+bx^2+cx+d$, for constants $a$, $b$, $c$, $d$ with $a\neq 0$.

**Zeros or Roots of a Function:** A zero (or root) of a function $f: \mathbb{R}\to\mathbb{R}$ is a number $x$ of the domain such that $f(x)=0$. A zero of a function is an element in the solution set of the equation $f(x)=0$. 
Figure 3.9: Sample vocabulary organizer.

My Definition
The slant of a line

Personal Association

Example
Slope is 1. Up 1 right 1.

Nonexample
Slope is not the y-intercept.

Characteristics/Attributes
Rate of change
\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
Rise over run
Positive – increasing to the right
Negative – decreasing to the right.

Teacher Definition
The ratio of the change in y compared to the change in x that expresses the rate of change.

Figure 5.9: Sample completed vocabulary organizer.

My Definition
Function that is not a constant or a line.

Personal Association

Example

Nonexample

Characteristics/Attributes
\[ ax^2 + bx + c = y \]
An equation that can be \( ax^2 + bx + c = y \) where a, b, and c are real numbers and \( a \neq 0 \).

Teacher Definition
\[ y = x^2 \]
## Vocabulary Notebook

<table>
<thead>
<tr>
<th>New Word and Illustration</th>
<th>Pronunciation</th>
<th>Definition in Your Own Words</th>
<th>Sentence and/or Connections</th>
</tr>
</thead>
<tbody>
<tr>
<td>Word: Sketch</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Definition:</td>
<td></td>
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<tr>
<td>Sentence:</td>
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<tr>
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<tr>
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<tr>
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<td></td>
</tr>
<tr>
<td>Definition:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sentence:</td>
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</tr>
</tbody>
</table>
Word Bank Cards; Cognate Glossaries

VOCABULARY SELF-RATING

Purpose  This activity alerts students to the key words they will learn and helps them plan and monitor their learning. It helps students be aware of what they know, and take responsibility for what they need to learn. The teacher adjusts lessons based on a quick review of students’ personal rating sheets.

Description  Students rate their knowledge of key vocabulary words before and after the *investigate* phase of the lesson. A student’s self-rating is personal; it may be shared with the teacher, but it is not graded. Students rate whether they know the word (K), do not know the word (DK), or are not sure (?) at three different points: before the lesson begins, after specific vocabulary instruction, and after instruction on mathematics content (at the end of the entire lesson).

Use  »  *Introduce* phase: The teacher pronounces each word and students rate their knowledge level. This alerts students to words they need to learn. A quick survey of completed columns alerts the teacher to which words to emphasize.

  »  *Investigate* phase: Students rate the words again after vocabulary instruction. Students see their growth and the teacher sees which words need more attention during the content lesson.

  »  *Summarize* phase: Students rate the words once again and see their growth, while the teacher sees which concepts need further discussion and what content related to the words may not be sufficiently understood.

Example  Students rate their knowledge of key words that are important in the day’s lesson on division of whole numbers.

<table>
<thead>
<tr>
<th>Vocabulary Self-rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name:</td>
</tr>
<tr>
<td>Lesson Topic:</td>
</tr>
<tr>
<td><strong>K:</strong> I am sure I know it</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Word (part of speech)</th>
<th>Before Lesson</th>
<th>After Vocabulary Discussion</th>
<th>After Lesson</th>
</tr>
</thead>
<tbody>
<tr>
<td>Divisor (n.)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Division (n.)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Quotient (n.)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Divided (v.)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Lesson Framing Strategies
In this lesson we will learn about:

- Proofs
- Theorems about Angles and Lines
- How theorems can be used as facts in proofs of other theorems
<table>
<thead>
<tr>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>TERM &amp; POLYNOMIAL FUNCTION</td>
</tr>
<tr>
<td>Constant function (Degree 0)</td>
</tr>
<tr>
<td>Linear function (Degree 1)</td>
</tr>
<tr>
<td>Quadratic function (Degree 2)</td>
</tr>
<tr>
<td>Cubic function (Degree 3)</td>
</tr>
</tbody>
</table>
Provide Concept Organizers to Complete as the Lesson Progresses

- Charts or Tables
- Diagrams
- (Partially Completed) Content Outlines
- T charts or Cornell Notes (2-column notes)
- Cumulative Summaries
- Daily Previews
- Focus Question—Cycle Back to Answer
Charts and Diagrams
5 principles for teaching ELLs

 Explicitly stated objectives in each lesson/module

 1. Identify and Communicate Content and Language Objectives
 2. Frontload the Lesson
 3. Provide Comprehensible Input
 4. Enable Language Production
 5. Assess for Content and Language Understanding

 Motivate
 Engage
 Success

 Opening activities connect to and assess prior knowledge

 Students express their understanding through speaking and writing activities; reflect on successful strategies

 Pair and group work promotes language practice in both listening/speaking and reading/writing activities

 Hands-on activities utilize multiple modes of instruction: visual, verbal, aural, kinesthetic
Interaction/Practice/Language
Use Strategies
Post it-Pile It - A technique to help encourage and increase participation

When posing questions to a class mixed with ELL, students who do not feel confident in their language skills may hesitate to respond. One way to increase participation in your classroom and help to include ELL in the discussion is to use the 'Post-it-Pile-It' technique. The teacher gives post-it notes to each student (they can vary in size depending on the length of the answer), poses a question and then has every student answer the question on the post it, the students then pile their answers under the question on the flip chart or board. This technique also makes the student responses anonymous, helping to encourage confidence in participation.
Practicing Language While Deepening Content Learning

- Help students use the core vocabulary you planned for the unit
- Make sure to extend students speaking ability: length and complexity
- Give sentence frames and starters

Activities that Allow Students to Deepen Content Knowledge while Using Language

- Show diagrams, equations that students have to explain to a partner
- Give charts, diagrams, visuals for students to complete (We do, then I do)
- Practice multiple choice items in quizzes and share responses; discuss why certain answers are the correct ones
Sentence Frames

Sentence frames. The following samples can be posted as a scaffold as students learn and practice their reasoning and oral participation skills.

- I think, because ........
- I predict, because ........
- I claim; my evidence is ........
- I agree with that ........
- My idea is similar/related to NAME’s idea ........
I learned/discovered/heard that ……..

<Name> explained to me……..

<Name> shared with me……..

We decided/agreed that ……..

Our group sees it differently, because …..

We have different observations/results. Some of us found that……..

One group member thinks that ……..

We had a different approach/idea/solution/answer.
### Figure 4.5 Discussion Sentence Starters

**Predicting**
- I guess/predict/imagine that...
- Based on..., I infer that...
- I hypothesize that...

**Expressing an Opinion**
- I think/believe that...
- In my opinion...
- It seems to me that...
- Based on my experience, I think...

**Asking for Clarification**
- What do you mean?
- Will you explain that again?
- How did you find your answer?

**Paraphrasing**
- So you are saying that...
- In other words, you think...
- What I hear you saying is...

**Soliciting a Response**
- What do you think?
- We haven’t heard from you yet.
- Do you agree?
- What is your solution? How did you get it?

**Acknowledging Ideas**
- My idea is similar to/related to ___’s idea.
- I agree with ___ that...
- My idea builds upon ___’s idea.

**Affirming**
- That’s an interesting idea.
- I hadn’t thought of that.
- I see what you mean.

**Holding the Floor**
- As I was saying...
- If I could finish my thought...
- What I was trying to say was...

**Reporting a Partner’s Idea**
- ___ shared with me that...
- ___ pointed out to me that...
- ___ emphasized that...
- ___ concluded that...

**Reporting a Group’s Idea**
- We decided/agreed that...
- We concluded that...
- Our group sees it differently.
- We had a different approach.

**Disagreeing**
- I don’t agree with you because...
- I got a different answer than you.
- I see it another way. I think...

**Offering a Suggestion**
- Maybe we could...
- What if we...
- Here’s something we might try.

---

Source: Adapted from Language Strategies for Active Classroom Participation (June 2007) with permission from Kate Kinsella. The document can be accessed as LanguageClassDiscussion.doc at http://www.sccoe.org/depts/ell/kinsella.asp. This webpage also lists many other “open access” documents that Kate Kinsella presents in her workshops.
Cooperative Learning/TPTs

- Assign groups/pairs
- If a Group; Assign roles within the groups
- Assign the task
- Provide appropriate amount of scaffolding
- Intervene to ensure full participation
- Keep students on track
Speaking Activities

- Reporting back
- Inside-Outside Circle
- Numbered Heads
- Snowball Questions
Cooperative Structures

- Numbered Heads Together
  - Students huddle to make sure all can respond, a number is called, the student with that number responds.
  - Paired Heads Together: Students in pairs huddle to make sure they both can respond, an “A” or “B” is called, the student with that letter responds.
Plan an Inside-Outside Circle for Our Unit to Give Students Practice
Snowball Technique

The Snowball cooperative learning strategy is a dynamic activity that allows the students to move around the classroom making use of their social and mastery skills. In this activity half of the students receive the questions on a colored sheet of paper and the other half of the students receive the answers on a different colored sheet of paper.

The students who have the same colored sheet of paper line up side by side and an imaginary line is drawn between the two groups of students. Once everyone is in line the teacher will ask the students to wad up their piece of paper and throw it across the imaginary line to the other side. The students then pick up one “snowball” and collaborate with others to find either their part of the question or the answer.

For example if the students were working on their multiplication facts the question would be a math problem and the answer would be a number. The students would then go around the classroom communicating with the other students until they find their match. You could also use this strategy to focus on vocabulary, review for a test, or match history events to their dates.
Providing Feedback to Learners: What and How to Provide Language Correction
Rules in Providing Feedback:

- Don’t just make incidental corrections; Show the student how English works or provide practice.
- Include the whole class in the practice activity.
- Use **Form-focused correction (FFI)**—show **one thing** you are noticing the students are doing (e.g. third person singular in present tense—leaving off the “s”—it go vs. it goes; not using have/has correctly); Keep on it till the production changes, help students self-edit for things you have taught them about English.
- Choose pronunciation errors that disturb meaning (substraction instead of subtraction; yust-ify vs. justify).
Resources for Math Teachers of ELLs
<table>
<thead>
<tr>
<th></th>
<th>Effective Strategies for Teaching Mathematics Content to English Learners</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Predictable Routines and Signals</td>
</tr>
<tr>
<td></td>
<td>Reducing anxiety non-language based classroom management</td>
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<tr>
<td>2</td>
<td>Advanced Organizers</td>
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<tr>
<td></td>
<td>Informing students of the learning goals</td>
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<tr>
<td>3</td>
<td>Preview / Review</td>
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<td></td>
<td>Building vocabulary and concepts to support understanding, summarizing, synthesizing</td>
</tr>
<tr>
<td>4</td>
<td>Academic Language Scaffolding</td>
</tr>
<tr>
<td></td>
<td>Supporting student use of language in academic settings (sentence frames)</td>
</tr>
<tr>
<td>5</td>
<td>Visual Scaffolding/Imaging</td>
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<td></td>
<td>Providing language support through visual images (Multiple representations)</td>
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<tr>
<td>6</td>
<td>Vocabulary Development Word Walls, Dictionaries</td>
</tr>
<tr>
<td></td>
<td>Displaying and organizing words for easy access: Frayer and JAN models, student dictionaries</td>
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<tr>
<td>7</td>
<td>Communication Practices &amp; Cooperative group work</td>
</tr>
<tr>
<td></td>
<td>Creating opportunities for verbal interaction about the mathematics</td>
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<tr>
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<td>Discussions: Pair-Share, small group, whole class</td>
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<tr>
<td>8</td>
<td>Modified teacher speech: Paraphrasing</td>
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<tr>
<td></td>
<td>Paraphrasing, repeat idea with correct vocabulary, adjust rate of speech, enunciate clearly (hundreds vs. hundredths)</td>
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<td>9</td>
<td>Leveled Questions</td>
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<tr>
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<td>Adjusting questioning strategies to the language and mathematics levels of students</td>
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<tr>
<td>10</td>
<td>Story Reenactment</td>
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<td></td>
<td>Using the “Act it out” strategy for a context or problem</td>
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<tr>
<td>11</td>
<td>Realia Strategies</td>
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<tr>
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<td>Connecting concept acquisition using real world objects</td>
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<tr>
<td>12</td>
<td>Manipulatives</td>
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<tr>
<td></td>
<td>Connecting concept acquisition using specially designed</td>
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<tr>
<td>13</td>
<td>Total Physical Response</td>
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<tr>
<td></td>
<td>Integrating movement into concept Acquisition</td>
</tr>
<tr>
<td>14</td>
<td>Modified Assessment</td>
</tr>
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<td></td>
<td>Less paper and pencil assessment</td>
</tr>
</tbody>
</table>
Contextual Clues for Mathematics

- Computers
- Calculators
- Technology Display
- Overheads
- Graphs
- Manipulatives
- Tables
- Patterns
- Symbolic Representations
- Gestures
- Realia
- Facial Expressions
- Props
- Visuals
- Acting Out Meaning

Modified from *Sheltered Instruction Across the Disciplines: Successful Teachers at Work*; Linda Sasser, Beth Winningham
<table>
<thead>
<tr>
<th>Strategy</th>
<th>Research</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Predictable Routines and</td>
<td>Even though your content will vary, follow a predictable routine and a stable schedule. Predictability in routine creates a sense of security for students who are experiencing a lot of change in their lives. (Perego &amp; Boyle, 1997)</td>
</tr>
<tr>
<td>Signals</td>
<td></td>
</tr>
<tr>
<td>2 Advanced Organizers</td>
<td>The final strategy is the use of <em>modeling, graphic organizers, and visuals</em>. The use of a variety of visual aids, including pictures, diagrams, and charts, helps all students—and especially ELL students—easily recognize essential information and its relationship to supporting ideas. Visuals make both the language and the content more accessible to students. (Alliance, 2005, p.2)</td>
</tr>
<tr>
<td>3 Preview/ Review</td>
<td>Results indicate that not only did the students in the preview-review group score significantly higher than the control and concurrent translation groups, the concurrent translation group scored the lowest of all three groups and improved slightly one week after treatment. These findings demonstrate positive implications for the use of strategies which build background knowledge as a means of teaching second language vocabulary to English learners. (Ulanoff &amp; Pucci, 1999, p.319)</td>
</tr>
<tr>
<td>4 Academic Language Scaffolding</td>
<td>Classroom instruction should support bilingual students engagement in conversations about mathematics that go beyond the translation of vocabulary and involve students in communicating about mathematical concepts. One of the goals of mathematics instruction for bilingual students should be to support all students, regardless of their proficiency in English, in participating in discussions about mathematical ideas. Teachers can move toward this goal by providing opportunities for bilingual students to participate in mathematical discussions and by learning to recognize the resources that bilingual students use to express mathematical ideas. (Moschkovich, 2002, p.208)</td>
</tr>
<tr>
<td></td>
<td><strong>Visual Scaffolding/ Imaging</strong></td>
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</tr>
<tr>
<td></td>
<td><strong>Vocabulary Development</strong></td>
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<tr>
<td></td>
<td><strong>Word Walls, Dictionaries</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Communication Practices/ Cooperative group work</strong></td>
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<tr>
<td>12</td>
<td>Manipulatives</td>
</tr>
<tr>
<td>13</td>
<td>Total Physical Response</td>
</tr>
</tbody>
</table>
One the basis of previous research, we believe that the following can be said with confidence at this time:

1. Translating test items from English into other languages does not appear to be an effective accommodation strategy when the students have studied the subject in a classroom where English is used. The language of assessment should match the students' primary language of instruction.

2. Some accommodations are more effective with certain student groups than with others, depending on background factors such as English reading proficiency and length of time in the United States.

3. The performance gap between English learners and other students has been narrowed by modifying the language of the test items to reduce the use of low-frequency vocabulary and complex language structures that are incidental to the content knowledge being assessed. This accommodation is effective; it is also valid, because it does not appear to affect the performance of English-proficient students.

4. Customized dictionaries can be an effective and valid alternative to commercial dictionaries; they have been found to help English learners while not affecting the scores of English-proficient students.

(Abedi et al., 2004, p. 17)
Focus: How does she get the student to understand the problem? (what scaffolds does she provide?)
How does she give the students practice with language?
Online PD for Teachers of ELLs

https://teachnkidslearn.com/u_course_cat/english-language-learners-ell/

### English Language Learners (ELL)

<table>
<thead>
<tr>
<th>ID</th>
<th>Course Name</th>
<th>Duration</th>
<th>Start Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>ELL Strand</td>
<td>Building Academic Language Skills and English Language Learners (ELL) through Dialogue, Discussion, and Discourse</td>
<td>Self-Paced</td>
<td></td>
</tr>
<tr>
<td>Differentiated Instruction</td>
<td>Strategies for Assessment-Driven Differentiated Instruction, Grades K-12</td>
<td>Self-Paced</td>
<td></td>
</tr>
<tr>
<td>Parent Involvement</td>
<td>Engaging Parents in Support of Student Learning, Grades K-12</td>
<td>Self-Paced</td>
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<tr>
<td>Student Discourse</td>
<td>Academic Discourse for All Students, Grades 6-12</td>
<td>Self-Paced</td>
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<td>Literacy Strand</td>
<td>Complex Textual Reading Made Easy, Grades 6-12</td>
<td>Self-Paced</td>
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<td>Literacy Strand</td>
<td>Building Academic Vocabulary and Deep Comprehension, Grades K-5</td>
<td>Self-Paced</td>
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<tr>
<td>Literacy Strand</td>
<td>Building Academic Vocabulary and Deep Comprehension, Grades 6-12</td>
<td>Self-Paced</td>
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<tr>
<td>Deepening Learning</td>
<td>Differentiated Instruction Driven by Assessments (Foundations)</td>
<td>Self-Paced</td>
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<tr>
<td>ELL Strand</td>
<td>Meeting ELL Students’ Needs in Today’s Classroom, Grades K-12</td>
<td>Self-Paced</td>
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<tr>
<td>ELL Strand</td>
<td>Academic Achievement For English Language Learners (ELLS)</td>
<td>Self-Paced</td>
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MAKING MATH ACCESSIBLE to English Language Learners

Practical Tips and Suggestions

Grades 9-12
Thank You!

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nancycloud2@gmail.com