Math Best Practices: Scaffolding through the Pillars of School Mathematics Grades 6 - 8

by
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NYS RBERN Resource Specialists
New York State Statewide Language RBERN at NYU
Citywide Mathematics Professional Development (Grades 6 – 8)
726 Broadway, New York, N.Y. 10003
Friday, November 16, 2018 (9:00 am – 2:00 pm)

AGENDA

9:00 - 11:30 am (Morning Session)
Greetings
1. Warm-ups
2. Student Data & PD Rationale
3. Best Practices: Scaffolding through the Pillars of School Mathematics (e.g., Socratic Method of Teaching & Learning, SMTL) (Inductive Reasoning & Deductive Reasoning)
   ▪ Scaffolding through Language (e.g., VVWA, Playlet, and Graphic Organizers)
   ▪ Scaffolding through Skills (Procedural Fluency)
   ▪ Scaffolding through Conceptual Understanding (Why $\pi = 3.14$?; Why $A = \pi r^2$?; Why $c^2 = a^2 + b^2$?; Why triangle angle sum = $180^\circ$?)
   ▪ Scaffolding for Problem Solving (Comparing/Contrasting Capacities; Formulating Equations; Mango Equations & Transformations)

LUNCH 11:30 am – 12:00 pm

12:00 – 2:00 pm (Afternoon Session)
4. Independent Group Work (Application of SMLT)
   ▪ Problem Solving, Presentations & Discussions
   ▪ Reflections/Implications for Classrooms
5. Evaluation
Place one of these numbers 1, 2, 3, 4, 5, and 6 inside each circle so that the **sum** of the numbers on each side of the “triangle” is 9.
Do not repeat a number. What pattern(s) have you observed?
WARM-UP

Place one of these numbers 1, 2, 3, 4, 5, and 6 inside each circle so that the sum of the numbers on the side of the “triangle” is 9. Do not repeat a number. What pattern(s) have you observed?

SUM 9
Place one of these numbers in each cell of the grid so that their sum equals 15, horizontally, vertically, or diagonally: 1, 2, 3, 4, 5, 6, 7, 8, 9. Do not repeat a number.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
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</tr>
</tbody>
</table>

**Magic Square**
15-SUM
<table>
<thead>
<tr>
<th>Self Assessment Statements</th>
<th>always</th>
<th>often</th>
<th>rarely</th>
<th>never</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. I elicit students’ prior knowledge using various means, i.e., realia, visuals, and stories.</td>
<td></td>
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<tr>
<td>2. I develop and deliver lessons that are rigorous and culturally-relevant to my class.</td>
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<tr>
<td>3. My classroom is print-rich, picture-rich, respectful, and conducive to learning.</td>
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<tr>
<td>4. I take time to know my students academically, socially and culturally.</td>
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<tr>
<td>5. I adopt a class configuration that allows easy access to all my students.</td>
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<tr>
<td>6. I use probing questions and provide time for responses and cues for scaffolding.</td>
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<tr>
<td>7. As I plan my lesson., I envision who would need one-on-one assistance.</td>
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<tr>
<td>8. I take notes of student learning and provide timely and formative feedback.</td>
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<tr>
<td>9. My lesson plan includes a teacher reflection box that will inform subsequent lessons.</td>
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<tr>
<td>11. I provide ample time for practice, reinforce efforts, and celebrate accomplishments.</td>
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<tr>
<td>12. I assess the concepts I have taught consistently with the NYS curriculum and standards.</td>
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<tr>
<td>13. I use the Frayer Model to elicit student prior knowledge and to assess learning (exit slip).</td>
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<tr>
<td>14. I am on the look out for professional development to further my knowledge on ELLs.</td>
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</tbody>
</table>
The 4 Pillars of 21st Century Mathematics

Organic-way Mathematics
(under study-Concordia University)

Concepts

Skills

Problems

Language
• Are conceptually packed.
• High-density of important words.
• Require up-and-down as well as left-to-right eye movements.
• Require reading-rate adjustment.
• Require multiple readings.
• Use numerous symbolic devices.
• Contain a great deal of technical language with precise meaning.
Decoding the language of math
Focal Concepts

1. Capacity / Volume & Proportion
2. Linear Equation / Function
3. Area & Geometric Transformation
Scaffolding Manipulatives Continuum

From most **Concrete** to most **Abstract**

MOST COMPREHENSIBLE INPUT

LEAST COMPREHENSIBLE INPUT

In quest for i+1 (Krashen)
LANGUAGE
Scaffolding
Visual-Verbal-Word Association (VVWA)
Visual-Verbal-Word Association
VVWA

Instructions:
1. Silently identify the symbols/shapes on #1 and #2.
2. Write both names in your notebook.
3. First complete the declarative sentence on page #1.
4. Then complete the interrogative sentence on #2.
I have ...

Who has...?
I have ...

Who has... ?
I have ... Who has... ?
I have ...  Who has... ?
I have ...

Who has... ?
I have ...

Who has...?
I have ...  

Who has...?
I have ...

Who has...?
I have ... Who has...?
I have ...

Who has...?
I have ...

Who has...?
I have ...

Who has...?
I have ... 

\{ \} 

∪

Who has...?
I have ...

Who has...?
I have ...

Who has...?
I have ...

Who has...?
I have ...

//

Who has...?

\[2x + 3 = 11\]
I have ... 2x + 3 = 11  

Who has...? 2x + 3
I have ...

$2x + 3$

Sorry!
We are out of shapes and symbols
Linguistic modalities covered by the activity
How?

<table>
<thead>
<tr>
<th>Listening</th>
<th>Speaking</th>
<th>Reading</th>
<th>Writing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</table>
### Mathematical Statements

<table>
<thead>
<tr>
<th>Mathematical Statements</th>
<th>Agree</th>
<th>Not sure</th>
<th>Disagree</th>
</tr>
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<tbody>
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<td>1. A square has the properties of both rectangle and rhombus.</td>
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<td>2. Any quadrilateral with 4 congruent sides is a square.</td>
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<tr>
<td>3. The diagonals of a rhombus are perpendicular.</td>
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<tr>
<td>4. The sum of the measures of all angles in a triangle is $108^\circ$.</td>
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<td></td>
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<tr>
<td>5. The quotient of the sum of $a$ and $b$, and their difference can be written as:</td>
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</tbody>
</table>
|    \[
    \frac{a+b}{a-b}
    \]                                                      |       |          |          |
| 6. “Three subtracted from five” can be written as “$3 - 5$”.                           |       |          |          |
| 7. The difference between the square of 3 and the double of 3 is 3.                     |       |          |          |
| 8. All improper fractions are greater than one.                                         |       |          |          |
| 9. In the linear equation defined as $y = mx + 3$, $x$ is the slope.                  |       |          |          |
| 10. Decimal 0.04785 is greater than decimal 0.4.                                        |       |          |          |
| 11. The diagonals of a rectangle are always perpendicular.                              |       |          |          |
| 12. Constant proportionality means slope ($m$) equation $y = mx + b$.                  |       |          |          |
**Word Etymology**

**Volume** = volumen (Latin for amount/size of roll/manuscript)

**Capacity** = capere (Latin for to take; capacitem = breadth, capability of holding much).

**Equation** = equation (Latin for an equalizing)

**Angle** = angulus (Latin for sharp bend. Also: ankle)

**Polygon** = polus (Greek for many) + gōnia (angle, corner)

**Acute** = acus (Latin for needle, sharp)

**Diagonal** = dia (Latin for to pass through or join) + bonus (angle)

**Diameter** = diametros; dia (Greek for pass through or join) + metron (Measure)

**Exponent** = exos (Latin for out of) + ponere (to place)

**Fraction** = fractio (Latin for breaking); frangere, to break

**Isosceles** = iso (Greek for the same) + skelos (legs)

**Polyhedron** = poli (Greek for many) + hedros (face)

**Mono/bi/trinomial** = mono = 1; bi = 2; tri = 3 + nomos = Greek for portion, part

**Geometry** = geo (Greek for Earth) + metria (measure)

**Slope** = sleubh (Latin for slip)

**Hypotenuse** = hypo (Greek for under) + tein (stretch)

**Congruent** = con (Latin for together) + ruere (fall); congruere (to come together)

**Chord** = chorde (Greek for string)

**Circle** = circus (Latin for circular race track)

**Kilo** = 1,000; hecto = 100; deca = 10; deci = 1/10; centi = 1/100; milli = 1/1000
Use a two-column chart to differentiate the one-meaning words from the multiple-meaning words. Explain your thinking.

<table>
<thead>
<tr>
<th>table</th>
<th>domain</th>
<th>exponent</th>
<th>dividend</th>
</tr>
</thead>
<tbody>
<tr>
<td>volume</td>
<td>root</td>
<td>hypotenuse</td>
<td>gross</td>
</tr>
<tr>
<td>power</td>
<td>bank</td>
<td>coefficient</td>
<td>terms</td>
</tr>
<tr>
<td>total</td>
<td>odd</td>
<td>equation</td>
<td>trapezoid</td>
</tr>
</tbody>
</table>
## Types of Words

<table>
<thead>
<tr>
<th>One-meaning Words</th>
<th>Multiple-meaning Words</th>
</tr>
</thead>
<tbody>
<tr>
<td>exponent</td>
<td>table</td>
</tr>
<tr>
<td>equation</td>
<td>dividend</td>
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<tr>
<td>trapezoid</td>
<td>root</td>
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<td>bank</td>
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<tr>
<td></td>
<td>odd</td>
</tr>
</tbody>
</table>
## Tiered Words
(Word Sophistication)

<table>
<thead>
<tr>
<th>Tier 1: Basic</th>
<th>Tier 2: Academic (across subjects)</th>
<th>Tier 3: Subject-Specific</th>
</tr>
</thead>
<tbody>
<tr>
<td>more</td>
<td>additive</td>
<td>addition</td>
</tr>
<tr>
<td>less</td>
<td>subtractive</td>
<td>subtraction</td>
</tr>
<tr>
<td>again</td>
<td>reproduction</td>
<td>multiplication</td>
</tr>
<tr>
<td>share</td>
<td>fragmentation</td>
<td>division</td>
</tr>
</tbody>
</table>
The syntax / sentence structure of math can be troublesome. Example: 3 \textit{subtracted from} 5 equals 2 can be written as $5 - 3 = 2$.

2. At times, key words can be deceiving. Use logic instead. Example:

\textit{John has 2 cats and 4 dogs. How many cats does he have in all?}
What math task(s) can you develop from this letter grid?

Match the words with their definitions on the next page. Then locate them on the grid.

TOTAL                      OPT
ROOT                       TOOL
INTEGER                    NAME
CHORD                      REPORT
ALL                        LOCUS
NO                         PERCENT
PROPORTION                 ADD
REST                       EPICENTER
SUM                        PRACTICAL
TOP                        NONAGON
FACTOR                     LAW
ORIGINAL

Math Glossary Puzzle
What math task(s) can you develop from this letter grid?

Match the words with their definitions on the next page. Then locate them on the grid.

TOTAL, OPT, ROOT, TOOL, INTEGER, NAME, CHORD, REPORT, ALL, LOCUS, NO, PERCENT, PROPORTION, ADD, REST, EPICENTER, SUM, PRACTICAL, TOP, NONAGON, FACTOR, LAW, ORIGINAL.
Math Glossary Puzzle
Word Definitions

_____________. Adjective for beginning
_____________. Synonym for principle or rule
_____________. A number that multiplies another
_____________. A polygon with nine sides
_____________. Antonym for bottom
_____________. Adjective for practice
_____________. Result of addition-Word ending with “M”
_____________. Point on earth at the center of a quake
Math Glossary Puzzle

Word Definitions

-_______. Put things together
-_______. Equality between two ratios
-_______. Meaning “for each 100”
-_______. Antonym for yes
-_______. Meaning “place or position”
-_______. Line segment connecting 2 points on the circumference
-_______. Conclusion from an investigation
Appellation (used to designate a person or thing)
Meaning “everything”
Base of a power. May also mean part of a plant
Instrument
Meaning “choose”
Synonym for sum
Any signed number. Example: −5, +6, etc.
Math Glossary Puzzle
Word Definitions

___________. Adjective for beginning
LAW
___________. Synonym for principle or rule
FACTOR
___________. A number that multiplies another
NONAGON
___________. A polygon with nine sides
TOP
___________. Antonym for bottom
PRACTICAL
___________. Adjective for practice
SUM
___________. Result of addition-Word ending with “M”
EPICENTER
___________. Point on earth at the center of a quake
ADD. Put things together
PROPORTION. Equality between two ratios
PERCENT. Meaning “for each 100”
NO. Antonym for yes
LOCUS. Meaning “place or position”
CHORD. Segment connecting 2 points on a circumference
REPORT. Conclusion from an investigation
### Math Glossary Puzzle

#### Definitions

<table>
<thead>
<tr>
<th>Name</th>
<th>Definition</th>
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</thead>
<tbody>
<tr>
<td>NAME</td>
<td>Appellation (used to designate a person or thing)</td>
</tr>
<tr>
<td>ALL</td>
<td>Meaning “everything”</td>
</tr>
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<td>ROOT</td>
<td>Base of a power. May also mean part of a plant</td>
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<td>TOOL</td>
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<td>OPT</td>
<td>Meaning “choose”</td>
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<tr>
<td>TOTAL</td>
<td>Synonym for sum</td>
</tr>
<tr>
<td>INTEGER</td>
<td>Any signed number. Example: $-5$, $+6$, etc.</td>
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</table>
Puzzle Website

To make your own, go to:

Puzzlemaker.discoveryeducation.com
The Frayer Model is a visual aid that helps students understand a concept by breaking it down into four sections: Definition, Facts/Characteristics, Examples, and Non-examples.
Capacity is the total amount that can be contained.

- 3-D shapes
- Base Area and Height

Examples:
- 2 liters
- \(\frac{3}{4}\) gallon

Non-examples:
- 0.5 m\(^2\)
- 6 in\(^3\)
Concept Map
<table>
<thead>
<tr>
<th>Prior Knowledge</th>
<th>What I Want to Know</th>
<th>What I Have Learned</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>What I <strong>know</strong></td>
<td>What I <strong>want to know</strong></td>
<td>What I <strong>have learned</strong></td>
</tr>
<tr>
<td>-----------------</td>
<td>--------------------------</td>
<td>-------------------------</td>
</tr>
<tr>
<td><strong>PRIOR-KNOWLEDGE</strong></td>
<td><strong>GOAL LESSON OBJECTIVE</strong></td>
<td><strong>OUTCOME</strong></td>
</tr>
<tr>
<td>I know that capacity is the total amount that can be contained in a 3-D object.</td>
<td>How is capacity different from volume?</td>
<td>I have learned that capacity is the amount of substance that a 3-D (solid) can contain based of measures from inside, whereas the volume is the space that the solid occupies based on measures from outside.</td>
</tr>
</tbody>
</table>
Venn Diagram

Capacity

- Space inside of containers
- Liter
- Etc.

Volume

- Height
- Base area
- 3-D shapes
- Weight
- Space outside of containers
- dm³
- Etc.
<table>
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<td></td>
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<td></td>
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<td>$\frac{a+b}{a-b}$</td>
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<td></td>
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</table>
1. I am a polyhedron 8 vertices, 12 edges, and 6 faces. Who am I?

2. I am a space figure with no vertex like the globe than fifteen. Who am I?

3. I am a number that is ten less than fifty. Who am I? Write my equation.

4. I am an even number. If you add six to me, and then subtract two, the result will be eighteen. Who am I?

5. I belong to a family of numbers having only two factors. Who am I?

6. In a fraction, I am the number on top of the bar. Who am I?

7. I am the borderline of enclosed shapes. Sometimes, students mistake me for area. Who am I?

8. I am the identity property for multiplication and division. Who am I?

9. I am the identity property for addition and subtraction. Who am I?

10. I am the inverse operation to addition. Who am I?

11. I am the inverse operation to division. Who am I?

12. I am simultaneously the square of 3 and one-half of 18. Who am I?
1. I am a quadrilateral with two pairs of parallel sides. Who am I?  
**RECTANGULAR PRISM**

2. I am a space figure with no vertex like the globe than fifteen. Who am I?  
**SPHERE**

3. I am a number that is ten less than fifty. Who am I? Write my equation.  
40  
n = 50 − 10

4. I am an even number. If you add six to me, and then subtract two, the result will be eighteen. Who am I?  
14

5. I belong to a family of numbers having only two factors. Who am I?  
**PRIME NUMBER**

6. In a fraction, I am the number on top of the bar. Who am I?  
**NUMERATOR**

7. I am the borderline of enclosed shapes. Sometimes, students mistake me for area. Who am I?  
**PERIMETER**

8. I am the identity property for multiplication and division. Who am I?  
1

9. I am the identity property for addition and subtraction. Who am I?  
0

10. I am the inverse operation to addition. Who am I?  
**SUBTRACTION**

11. I am the inverse operation to division. Who am I?  
**MULTIPLICATION**

12. I am simultaneously the square of 3 and one-half of 18. Who am I?  
9
Examine the next slide. Write a relevant caption underneath each set of images, and formulate a short story problem.
Story Boarding

1. Car
2. Parking meter
3. Market
4. Car
5. People eating
6. Jars of food
Narrator: Welcome Pam and Antonio. Pam is a customer who wants some apples to make applesauce. Antonio is a store keeper. Today, Pam walks into Antonio’s Fruit Store, and starts the conversation.

Pam: Good morning, Antonio!
Antonio: Good morning, Pam! How can I help you today?
Pam: I need some apples to make applesauce.
Antonio: That’s a good idea. Today, they are fresh and delicious.
Pam: How do you sell the red apples?
Antonio: Two for three.
Pam: All right! I take five

Narrator: Freeze! How much does Pam owe for the red apples?
Antonio: Umm! Let’s see... $______.
Pam: Ok! I need green apples, too.
Antonio: That’s a good idea. Today, they’re sweet and
Pam: Is it the same price?
Antonio: Nope!
Pam: How do you sell them?
Antonio: Three for two.
Pam: All right! I take twelve.

Narrator: Freeze! How much does Pam owe for the green apples?
Antonio: Umm! Let’s see... $______.
Pam: Ok. That’s all.
Pam: Here is a twenty-dollar bill. Keep the change!

Narrator: Freeze! How much change (if any) has Pam left behind? Explain.
Antonio: Umm! Let’s see... ____________________________. Thank you.
Narrator: Ladies and gentlemen, please welcome Jeanne and Mary. Jeanne is a young lady who loves mango juice. Mary is a peddler who wants to sell as many mangoes as possible. This morning, out goes Jeanne in search of her favorite fruit. As she crosses a street corner, a soft voice breaks the silence in her mind.

Mary: My dear beautiful lady, would you come and check my mangoes out?
Jeanne: Ok. No problem. Indeed, I do need some mangoes to make some juice.
Mary: That’s a good idea. Today they’re fresh and delicious.
Jeanne: How do you sell these Francique mangoes?
Mary: Five for three.
Jeanne: Ok. No problem. I’m taking three dozen.

Narrator: Freeze! How much does Jeanne owe for the Francis mangoes?
Mary: Mmm! Let’s see... $__________.
Jeanne: Ok! I also need some Cinnamon mangoes.
Mary: That’s a good idea. Today they’re sweet and juicy.
Jeanne: Is it the same price?
Mary: Nope! These are special. Five for four.
Jeanne: Ok. No problem. I’m taking three and a half dozen.

Narrator: Freeze! How much does Jeanne owe for these Cinnamon mangoes?
Mary: Mmm! Let’s see... $__________

Narrator: Freeze! How much does Jeanne owe in all?
Mary: Mmm! Let’s see... $__________
Jeanne: Ok. No problem. Here’s a crispy fifty-dollar bill. Keep the change!

Narrator: Freeze! How much change (if any) has Jeanne left for Mary? Explain.
Mary: Mmm! Let’s see... $__________
Mango Juice Sample Questions

1. Who is Mary? Why does she want the mangoes for?

2. Who is Jeanne? What’s her desire?

3. What is a peddler?

4. Who begins the conversation? Jeanne or Mary? Cite some textual evidence.

5. Which mango brand is more expensive? By how much?

6. What adjectives are used to determine the Francique mangoes?

7. What adjectives are used to describe the Cinnamon mangoes?

8. Why does Jeanne use the determinant “crispy” to characterize her 5-dollar bill?

9. How much change has Jeanne left for Mary? How do you know?

Revelation of a Polygon

Blessed am I with some special qualities! Mathematicians would say unique properties. ‘Cause my two diagonals are perpendicular, Thus definitely anointing me in particular.

Above all, I am a full-fledged Polygon. From my vertices, multiple sides can be drawn. Moreover, I do inherit traits of Parallelogram, ‘Cause parallelism sprawls all over my diagram.

Any clone of mine inevitably is a Rectangle, ‘Cause each angle is 90 degrees, a right angle. But don’t be fooled! All rectangle copies Don’t necessarily possess my qualities.

To some extent, I do look like a Rhomboid. Also, I share a thing or two with a Trapezoid. Definitely, I am a Rhombus; my sides are the same. Oh, boy! Can’t you still guess my name.

June 2012
Blessed am I with some special qualities!
Mathematicians would say unique properties.
‘Cause my two **diagonals** are **perpendicular**,
Thus definitely anointing me in particular.

Above all, I am a **full-fledged** Polygon.
From my **vertices**, multiple sides can be drawn.
Moreover, I do inherit traits of Parallelogram,
“Cause parallelism sprawls all over my diagram.

Any clone of mine **inevitably** is a Rectangle,
“Cause each angle is 90 degrees, a **right angle**.
But don’t be fooled! All rectangle copies
Don’t necessarily possess my qualities.

To some extent, I do look like a **Rhomboid**.
Also, I share a thing or two with a **Trapezoid**.
Definitely, I am a **Rhombus**; my sides are the same.
Oh, boy! Can’t you still guess my name.
1. Which specific polygon is the poem talking about? How do you know? Cite textual evidence.

2. Underline the bolded words and explain their meanings.

3. What does “to some extent” mean?

4. Underline all adverbs in the poem and write down their meanings.

5. Rewrite each stanza in your own words.

6. Formulate your own questions.
Two days following her birthday party, Marie returned into the neighborhood store where she had purchased the beverages. She wanted to redeem the bottles. Once there, she dashed to the machine and began inserting the empties. When she finished, she pressed a button, and a total of one dozen nickels and dimes chimed into a shiny metal tray at the bottom. The monetary mixture amounted to one dollar. Astonishingly, Marie realized that there were twice as many dimes as nickels.
Sample Questions

• Who went to the neighborhood store?
• When did Marie go to the store?
• What did Marie do after her birthday?
• Where did Marie insert the empty bottles?
• How did Marie go to the store?
• Why did Marie return to the store?
• Name one synonym for “beverages.”
• What does the word “redeem” mean?
• What is the value of one dime?
• What does the word “mixture” signify?
• How do you interpret “twice as many?”
• What is the value of one nickel?
• What is “one dozen?”
• Why does the author use the perfect past tense “had purchased” instead of the simple past tense “purchased?”
Learning Log

- Journal entries (on what has been learned) specified by the teachers
- Not all journal entries are learning logs.
SKILLS
Scaffolding

Computational Fluency = Speed + Accuracy
12 \times 13 = 156
$24 \times 35 = 840$
$30 \times 245 = 7,350$
Haitian Division

480 ÷ 15 = 32

dividend  divisor
480  15
030  
000  

remainder
15
32
quotient
Metric System Prefixes

kilo = 1,000
hecto = 100
deca = 10

Standard = 1 (meter, gram, liter)

deci = \(\frac{1}{10}\)

centi = \(\frac{1}{100}\)
milli = \(\frac{1}{1000}\)
### System Metric Table for Length

<table>
<thead>
<tr>
<th>km</th>
<th>hm</th>
<th>dam</th>
<th>m</th>
<th>dm</th>
<th>cm</th>
<th>mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thousands</td>
<td>Hundreds</td>
<td>Tens</td>
<td>Ones</td>
<td>Tenths</td>
<td>Hundredths</td>
<td>Thousandths</td>
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<tr>
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<td>10</td>
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<td>.1</td>
<td>.01</td>
<td>.001</td>
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<tr>
<td>7</td>
<td>4</td>
<td>6</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7465 m = **74.65** hm
# System Metric Table for Area

<table>
<thead>
<tr>
<th>km²</th>
<th>hm²</th>
<th>dam²</th>
<th>m²</th>
<th>dm²</th>
<th>cm²</th>
<th>mm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 5</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
5 \text{ km}^2 = 5,000,000 \text{ m}^2
\]

Note: 1 km² = 1,000,000 m²
## System Metric Table for Volume/Capacity

<table>
<thead>
<tr>
<th>km³</th>
<th>hm³</th>
<th>dam³</th>
<th>m³</th>
<th>dm³</th>
<th>cm³</th>
<th>mm³</th>
</tr>
</thead>
<tbody>
<tr>
<td>005</td>
<td>000</td>
<td>000</td>
<td>000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ 5 \, \text{km}^3 = 5,000,000,000 \, \text{m}^3 \]

Note: 1 km³ = 1,000,000,000 m³
Scaffolding for Conceptual Understanding

The Socratic Method
Productive talk is:

- **Deep**: The specific idea being discussed is central to the lesson, presented in interconnected ways, and engages students’ analytical thinking.

- **Sustained**: One student’s statement is followed by another student’s response, which extends, refutes, or questions what was first said.

- **Student-controlled**: Students (not teachers) control what they say. But teachers set the parameters for interactions, sometimes framing questions that start the conversation. These questions are intended to communicate new related ideas, propose counter ideas or counter-examples, and generally enhance the discussion.
The Socratic Method of Teaching & Learning

1. Hypothesis / Claim / Statement

2. Clarification of the Hypothesis

3. Experimentation / Proof

4. Validation / Adjustment / Rejection of hypothesis
Clarifying Questions

- Factual questions of fact clarifying the dilemma, and providing the nuts and bolts to probing questions

Examples

1. Is this what you said...?
2. Did I hear you say...?
3. Did I understand you when you said...?
4. What criteria did you use to...?
5. What’s another way you might...?
6. Did I paraphrase what you said correctly?

Adapted from the Iowa Peace Institute Message
Probing Questions

Inquisitive questions intended to help the presenter think more deeply about the issue at hand.

Examples

1. Why do you think this is the case?
2. What do you think would happen if...?
3. What sort of impact do you think...?
4. How did you decide...?
5. How did you conclude...?
6. How did you determine...?
7. What is the connection between... and...?
8. What if the opposite were true? Then what?
Why $3 \div \frac{1}{2} = 6$?

- What does it mean to divide 3 by $\frac{1}{2}$?
- What real-life situation would require the division above?
Answer: 6

Six halves \(\frac{1}{2}\). Not mangoes.
Why multiply by the reciprocal when dividing fractions?

\[
\frac{3}{1} \div \frac{1}{2} = \frac{3 \times 2}{1 \times 1} = \frac{6}{1} = 6
\]
\[ \frac{3}{4} \div \frac{1}{8} = 6 \]

Answer: There are 6 eighths in three-fourths.
Why $10^0 = 1$?

- **Multiplication of powers:** $10^3 \times 10^2 = 10^{(3+2)} = 10^5 = 10,000$

- **Division of powers:** $10^3 \div 10^2 = 10^{(3-2)}$
  
  $10^3 \div 10^2 = 10^1 = 10$

The same principle applies to:

- $10^3 \div 10^3 = 10^{(3-3)}$
  
  $10^3 \div 10^3 = 10^0$

- $10^3 \div 10^3 = 1$

So, $10^0 = 1$
Why the sum of the measures of all angles inside a triangle equals 180°?
Why $\pi = 3.14$?

The Meaning of Pi.

Use tape measurement to determine the circumference and the diameter of your given circle. Then submit your findings to the class to complete the collaborative table (next slide).
<table>
<thead>
<tr>
<th>Circumference</th>
<th>Diameter</th>
<th>Ratio Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Why Area of Circle = $\pi r^2$?
\[
A = r\left(\frac{1}{2}C\right)
\]
\[
A = r\left(\frac{1}{2}d\pi\right)
\]
\[
A = r\left(\frac{1}{2}2r\pi\right)
\]

\[
A = r\left(r\pi\right)
\]

\[
A = \pi r^2
\]
Pythagorean Theorem

Centre de Ressources Éducatives du Bas-Artibonite (CREBA) teachers in Haiti applying (real-life) the Pythagorean Formula at a training session in the City of Saint-Marc. August 2018
Volume vs. Capacity

What is the difference between **Volume** and **Capacity**?
Volume v. Capacity

- Which cup has the greater volume? Why?
- Which cup has the greater capacity? Why?

Blue Cup

White Cup
Exploring Prism

A rectangular prism can be viewed as is a pile of rectangles. The collection forms a structure having a length, a width, and a height.
Exploring Cylinder

A cylinder can be viewed as a pile of circles. The collection forms a structure having a diameter and a height.
Key Formulae

- $1 \text{ dm}^3 = 1 \text{ kg} = 1 \text{ Liter}$
- $1 \text{ kg} = 2.2 \text{ Lbs.}$
- $1 \text{ cm}^3 = 1 \text{ gram} = 1 \text{ milliliter}$
- $1 \text{ kg} = 2.2 \text{ Lbs.}$

- $F^\circ = 1.8C + 32$
- $C^\circ = \frac{5}{9} (F - 32)$
Capacity

Centre de Ressources Éducatives du Bas-Artibonite (CREBA) teachers in Haiti exploring (real-life) the notion of capacity of a cylinder at a training session in the City of Saint-Marc and Gonaives. August 2018
Two Cylinders

(Application of the Socratic Method)

Which of the two cylinders has a greater capacity? The shorter one or the taller one? How do you know?
Formulating Equations or Functions From Patterns
Generating Equations or Functions
### Input-Output/Table of Value

<table>
<thead>
<tr>
<th>t # of Triangles</th>
<th>s # of Sticks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>10</td>
<td>21</td>
</tr>
<tr>
<td>1,500</td>
<td>3,001</td>
</tr>
</tbody>
</table>
Formulating Equations or Functions

\[ s = 2t + 1 \]

or

\[ f(t) = 2t + 1 \]

\[ y = 2x + 1 \]

or

\[ f(x) = 2x + 1 \]
Graphing Linear Equation

The equation is $S = 2t + 1$.

- $(0, 0)$
- $(1, 3)$
- $(2, 5)$
- $(3, 7)$
- $(4, 9)$
- $(5, 11)$
- $(6, 13)$
- $(10, 21)$
Equations or Functions with Color Tiles
Market 1

Taxi $6

1 for $0.50

Market 2

No Taxi

1 for $2
The Variables or Coordinates

Independent Variable $x$

Dependent Variable $y$
\[ y = 0.5x + 6 \quad (1) \]
\[ y = 2x \quad (2) \]

X mangoes \quad Y dollars
### Function Machine
#### Double-Entry Input-output Table

<table>
<thead>
<tr>
<th>Quantity of Mangoes</th>
<th>MARKET 1 ( y = 0.5x + 6 )</th>
<th>MARKET 2 ( y = 2x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6.00</td>
<td>0.00</td>
</tr>
<tr>
<td>1</td>
<td>6.50</td>
<td>2.00</td>
</tr>
<tr>
<td>2</td>
<td>7.00</td>
<td>4.00</td>
</tr>
<tr>
<td>3</td>
<td>7.50</td>
<td>6.00</td>
</tr>
<tr>
<td>4</td>
<td><strong>8.00</strong></td>
<td><strong>8.00</strong></td>
</tr>
<tr>
<td>5</td>
<td>8.50</td>
<td>10.00</td>
</tr>
<tr>
<td>6</td>
<td>9.00</td>
<td>12.00</td>
</tr>
<tr>
<td>7</td>
<td>9.50</td>
<td>14.00</td>
</tr>
<tr>
<td>8</td>
<td>10.00</td>
<td>16.00</td>
</tr>
</tbody>
</table>
Extending Understanding
Graph Analysis: Area

Y-intercept (0,6)
X-axis (0,0)
Y-axis 8
X-axis 12
Solution of this system of equations

Market 1
Market 2

Total Expense
Number of Mangoes

Area?

y = 2x
y = 0.5x + 6

Solution of this system of equations
Equations/Functions
Additional Activities

1. The grain of rice story: One grain of rice being doubled every day
2. Stacking up identical books from the floor
3. Stacking up identical books mounted on 6-inch pedestal
4. Cell phone sales competition
5. Mowing the lawn with per-hour pay
6. Shoveling the snow for a $5-sandwich upfront and per-hour pay
7. Diameter = 2r
8. Circumference = 3.14d
9. Area = 3.14r^2
10. Fahrenheit = 1.8C + 32
Review: Scaffolding Strategies & Frameworks

1. Verbal-Visual-Word Association (VVWA)
2. Concept Map
3. Sentence Starters / prompts
4. Word Etymology & Vocabulary
5. KWL Chart
6. Anticipatory Guide
7. Frayer Model
8. Math Glossary Puzzle
9. Flow Chart / Table / Diagram
10. Math Poetry
11. Socratic Method for Teaching & Learning
12. Project-based Learning
13. Organic-way Math (under study)
14. Math Games (e.g., ORGABEZ)
15. Etc.
PM Activities
Group Tasks & Presentations
12:00 – 1:30 PM
# New Library Proposal Table

<table>
<thead>
<tr>
<th>Room</th>
<th>Area</th>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$400</td>
</tr>
<tr>
<td>Info</td>
<td></td>
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</tr>
<tr>
<td>English</td>
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<td></td>
</tr>
<tr>
<td>Science</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Art</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Social Studies</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Library</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
A

Bottle taking in 1/4 gallon of water per minute.

B

5-gallon water Bottle to be emptied in 20 minutes

The 5-gallon Bottles

Same amount of water? When?

5-gallon water Bottle taking in 1/4 gallon of water per minute.
Gas Tank

After traveling a distance of 90 miles, a truck with a full tank has 40 gallons of gas left. The truck continued its journey. After cruising a distance of 360 miles, the vehicle had only 10 gallons of gas left in the tank.

1. How much more (longest) distance could the truck travel before refueling?
2. How much gas fills the tank?

Explain. Use various strategies.
Coordinated Polygons

1. Plot the points:
   A(3,3); B(5,3); E(4,0); F(2,0);
   D(6,0); C(6,6).

2. Connect the points to create the following polygons: ABEF, BDE, BCD, and ABCDF.

3. Find the area of each polygon and complete the table.

4. Write your observation of the table, especially what amazes you the most.
<table>
<thead>
<tr>
<th>Polygon</th>
<th>Area</th>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABEF</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$500</td>
</tr>
<tr>
<td>BDE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BCD</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ABCDF</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Observation:

---

---

---

---

---

---
The Better Buy

Use cm ruler to investigate which of the two containers filled with the same brand of substance is the better buy. Prove it theoretically (formula) and empirically (with rice).
Mango Transaction

Universal Design For Learning

1. Multiple forms of representation
2. Multiple forms of engagement
3. Multiple forms of expression
Some Research-based 21\textsuperscript{st} Century Models

- Standards-based Model
- Constructivist Model
- Discovery Model
- Inquiry-based Model
- Project-based Model
- Japanese Model (Neriage)
- Chinese Model (Bantu)
- Organic-way Model (under study)
- Etc.
Thank You
References

Organic-way Mathematics Consulting & Publishing, LLC.