New York State Statewide Language RBERN at NYU
Citywide Mathematics Professional Development

726 Broadway, New York, N.Y. 10003
Friday, November 2, 2018

Math Best Practices:
Designing & Delivering Effective Lessons for English Language Learners (ELLs)
Grades 3 - 5

by
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NYS RBERN Resource Specialists
Place one of these numbers 1, 2, 3, 4, 5, and 6 inside each circle so that the sum of the numbers on each side of the “triangle” is 9. Do not repeat a number. What pattern(s) have you observed?
Place one of these numbers in each cell of the grid so that their sum equal 15, horizontally, vertically, or diagonally: 1, 2, 3, 4, 5, 6, 7, 8, 9. Do not repeat a number.

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<tr>
<td>Self Assessment Statements</td>
<td>always</td>
<td>often</td>
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<td>-------------------------------------------------------------------------------------------</td>
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<tr>
<td>1. I elicit students’ prior knowledge using various means, i.e., realia, visuals, and stories.</td>
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<td>2. I develop and deliver lessons that are rigorous and culturally-relevant to my class.</td>
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<td>3. My classroom is print-rich, picture-rich, respectful, and conducive to learning.</td>
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<td>4. I take time to know my students academically, socially and culturally.</td>
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<td>5. I adopt a class configuration that allows easy access to all my students.</td>
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<td>6. I use probing questions and provide time for responses and cues for scaffolding.</td>
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<td>7. I envision which individual students needing one-on-one assistance as I plan my lesson.</td>
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<td>8. I take notes of student learning and provide timely and formative feedback.</td>
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<td>9. My lesson plan includes a teacher reflection box that will guide subsequent lessons.</td>
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<td>11. I provide ample time for practice, reinforce efforts, and celebrate accomplishments.</td>
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<td>12. I assess what I have taught consistent with the current NYS curriculum and standards.</td>
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<tr>
<td>13. I use the Frayer Model to elicit student prior knowledge and to assess learning (exit slip).</td>
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<tr>
<td>14. I am on the look out for professional development to further my knowledge on ELLs.</td>
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</table>
### All NYS Student Demographics

#### Math Gr. 3-8 Results

<table>
<thead>
<tr>
<th>Demographics</th>
<th>2018 %</th>
<th>2017 %</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asian</td>
<td>71.2</td>
<td>67.2</td>
<td>+4.0</td>
</tr>
<tr>
<td>White</td>
<td>54.2</td>
<td>50.4</td>
<td>+3.8</td>
</tr>
<tr>
<td>Amer. Indian</td>
<td>36.3</td>
<td>31.3</td>
<td>+5.0</td>
</tr>
<tr>
<td>Hispanic</td>
<td>31.8</td>
<td>27</td>
<td>+4.8</td>
</tr>
<tr>
<td>Black</td>
<td>29.3</td>
<td>24.4</td>
<td>+4.9</td>
</tr>
<tr>
<td>ELLs</td>
<td>16.1</td>
<td>13.2</td>
<td>+2.9</td>
</tr>
<tr>
<td>SWD</td>
<td>14.6</td>
<td>11.4</td>
<td>+2.2</td>
</tr>
</tbody>
</table>
Grade 3 Overview
Instructional time should focus on four areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing polygons based on the number of sides and vertices.

Please note that while every standard/topic in the grade level has not been included in this overview (and subsequent ones), all standards should be included in instruction.
Grade 4 Overview

Instructional time should focus on three areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends;

(2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers;

(3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.
Grade 5 Overview

In Grade 5, instructional time should focus on three areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimals into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; (3) developing understanding of volume.
Promoting student achievement through:

1. Scaffolding for computational fluency, conceptual understanding, problem solving, and linguistic abilities (Workshop Model)

2. Fostering student creativity / innovative ideas (Inquiry-based Learning/Constructivism/Project-based Model, etc.)
Some Research-based 21st Century Models

- Standards-based Model
- Constructivist Model
- Discovery Model
- Inquiry-based Model
- Project-based Model
- Japanese Model (Neriage)
- Chinese Model (Bantu)
- Organic-way Model (under study)
- Etc.
The 4 Pillars of 21st Century Math

- Concepts
- Problems
- Skills
- Language
Mathematics Discourse Features (Dr. Nancy Cloud)

- conceptually packed.
- high-density of important words.
- require up-and-down as well as left-to-right eye movements.
- require reading-rate adjustment.
- require multiple readings.
- use numerous symbolic devices.
- contain a great deal of technical language with precise meaning.
Visual-Verbal-Word Association (VVWA)

(ON INDEX CARDS)
Angle = angulus (Latin for sharp bend. Also: ankle)
Polygon = polus (Greek for many) + gōnia (angle, corner)
Acute = acus (Latin for needle, sharp)
Diagonal = dia (Latin for to pass through or join) + gonus (angle)
Diameter = diametros; dia (Greek for pass through or join) + metron (Measure)
Exponent = exos (Latin for out of) + ponere (to place)
Fraction = fractio (Latin for breaking); frangere, to break
Isosceles = iso (Greek for the same) + skelos (legs)
Polyhedron = poli (Greek for many) + hedros (face)
Mono/bi/trinomial = mono = 1; bi = 2; tri = 3 + nomos = Greek for portion, part
Geometry = geo (Greek for Earth) + metria (measure)
Slope = sleubh (Latin for slip)
Hypotenuse = hypo (Greek for under) + tein (stretch)
Congruent = con (Latin for together) + ruere (fall); congruere (to come together)
Chord = chorde (Greek for string)
Circle = circus (Latin for circular race track)
Kilo = 1,000; hecto = 100; deca = 10; deci = 1/10; centi = 1/100; milli = 1/1000
### Vocabulary

#### Types of Words

Use a two-column chart to differentiate the one-meaning words from the multiple-meaning words. Explain your thinking.

<table>
<thead>
<tr>
<th>table</th>
<th>domain</th>
<th>exponent</th>
<th>dividend</th>
</tr>
</thead>
<tbody>
<tr>
<td>volume</td>
<td>root</td>
<td>hypotenuse</td>
<td>gross</td>
</tr>
<tr>
<td>power</td>
<td>bank</td>
<td>coefficient</td>
<td>terms</td>
</tr>
<tr>
<td>total</td>
<td>odd</td>
<td>equation</td>
<td>trapezoid</td>
</tr>
</tbody>
</table>
The syntax / sentence structure of math can be troublesome. Example: 3 subtracted from 5 equals 2 can be written as $5 - 3 = 2$.

2. At times, key words can be deceiving. Use logic instead. Example:

*John has 2 cats and 4 dogs. How many cats does he have in all?*
What math task(s) can you develop from this letter grid?

**Math Glossary Puzzle**

Match the words with the definitions on the next page. Then locate them on the grid.

<table>
<thead>
<tr>
<th>TOTAL</th>
<th>OPT</th>
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<tbody>
<tr>
<td>ROOT</td>
<td>TOOL</td>
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<tr>
<td>INTEGER</td>
<td>NAME</td>
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<td>CHORD</td>
<td>REPORT</td>
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<tr>
<td>ALL</td>
<td>LOCUS</td>
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<tr>
<td>NO</td>
<td>PERCENT</td>
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<td>PROPORTION</td>
<td>ADD</td>
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<td>REST</td>
<td>EPICENTER</td>
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<td>SUM</td>
<td>PRACTICAL</td>
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<td>TOP</td>
<td>NONAGON</td>
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<td>FACTOR</td>
<td>LAW</td>
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<td>ORIGINAL</td>
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<thead>
<tr>
<th>P R O P O R T I O N</th>
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<tbody>
<tr>
<td>E E R P P O S U M O</td>
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<td>R S I R T O T A L N</td>
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<tr>
<td>C T G A S T O O L A</td>
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<tr>
<td>E P I C E N T E R G</td>
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<tr>
<td>N I N T E G E R E O</td>
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<tr>
<td>T B A I N O T O P N</td>
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<tr>
<td>A L L C H O R D O A</td>
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<tr>
<td>D A F A C T O R R M</td>
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<tr>
<td>D W Y L O C U S T E</td>
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</tbody>
</table>
Math Glossary Puzzle
Word Definitions

___________. Adjective for beginning
___________. Synonym for principle or rule
___________. A number that multiplies with another
___________. A polygon with nine sides
___________. Antonym for bottom
___________. Adjective for practice
___________. Result of addition-Word ending with “M”
___________. Point on earth at the center of a quake
_________. Put things together
_________. Equality between two ratios
_________. Meaning “for each 100”
_________. Antonym for yes
_________. Meaning “place or position”
_________. Line segment connecting 2 points on the circumference
_________. Conclusion from an investigation
Puzzle Website

To make your own, go to:

Puzzlemaker.discoveryeducation.com
Productive talk is:

- **Deep**: The specific idea being discussed is central to the lesson, presented in interconnected ways, and engages students’ analytical thinking.

- **Sustained**: One student’s statement is followed by another student’s response, which extends, refutes, or questions what was first said.

- **Student-controlled**: Students (not teachers) control what they say. But teachers set the parameters for interactions, sometimes framing questions that start the conversation. These questions are intended to communicate new related ideas, propose counter ideas or counter-examples, and generally enhance the discussion.
The Socratic Method of Teaching & Learning

1. Hypothesis / Claim / Statement

2. Clarification of the Hypothesis

3. Experimentation / Proof

4. Validation / Adjustment / Rejection of hypothesis
Frayer Model
Simple Concept Map
Free-Association Brainstorming
<table>
<thead>
<tr>
<th>PRIOR KNOWLEDGE</th>
<th>GOAL</th>
<th>OUTCOME</th>
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<tr>
<td>Mathematical Statements</td>
<td>Agree</td>
<td>Not sure</td>
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<td>---------------------------------------------------------------------------------------</td>
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<tr>
<td>1. A square is a quadrilateral.</td>
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<td>2. A rectangle has only two right angles.</td>
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<tr>
<td>3. Multiplication is repeated additions of the same addends.</td>
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<td>4. The sum of the measures of all angles in a triangle is 108°.</td>
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<tr>
<td>5. Twenty-five percent is comparable to 0.25 or ¼.</td>
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<tr>
<td>6. “Three subtracted from five” can be written as “3 – 5”.</td>
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<tr>
<td>7. The perimeter is the borderline of the polygon.</td>
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<tr>
<td>8. A fraction has a numerator and a denominator.</td>
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<td>9. The area is the inside of the polygon.</td>
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<tr>
<td>10. Decimal 0.04785 is greater than 0.4.</td>
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</table>
1. I am a quadrilateral with two pairs of parallel sides. Who am I?

2. I am a number that is three more than fifteen. Who am I?

3. I am a number that is ten less than fifty. Who am I?

4. I am an even number. If you add six to me, and then subtract two, the result will be eighteen. Who am I?

5. I belong to a family of numbers having only two factors. Who am I?

6. In a fraction, I am the number on top of the bar. Who am I?

7. I am the borderline of enclosed shapes. Sometimes, students mistake me for area. Who am I?

8. I am the identity property for multiplication and division. Who am I?

9. I am the identity property for addition and subtraction. Who am I?

10. I am the inverse operation to addition. Who am I?

11. I am the inverse operation to division. Who am I?

12. I am the square of 3 and at the same times one-half of 18. Who am I?
Story Boarding

Examine the next slide.
Write a relevant caption underneath each set of images, and formulate a short story problem.
Welcome Pam and Antonio. Pam is a customer who wants some apples to make applesauce. Antonio is a store keeper. Today, Pam walks into Antonio’s Fruit Store, and starts the conversation.

Pam: Good morning, Antonio!
Antonio: Good morning, Pam! How can I help you today?
Pam: I need some apples to make applesauce.
Antonio: That’s a good idea. Today, they are fresh and delicious.
Pam: How do you sell the red apples?
Antonio: One for forty cents.
Pam: All right! I take ten.

Freeze! How much does Pam owe for the red apples?

Antonio: Umm! Let’s see... $______.
Pam: Ok! I need green apples, too.
Antonio: That’s a good idea. Today, they’re sweet and juicy.
Pam: Is it the same price?
Antonio: Nope!
Pam: How do you sell them?
Antonio: Two for one dollar.
Pam: All right! I take fourteen.

Freeze! How much does Pam owe for the green apples?

Antonio: Umm! Let’s see... $______.
Pam: Ok. That’s all.
Pam: Here is a crispy ten-dollar bill. Keep the change!

Freeze! How much change (if any) has Pam left behind? Explain.

Antonio: Umm! Let’s see... _________________________________. Thank you.
Narrator: Ladies and gentlemen, please welcome Jeanne and Mary. Jeanne is a young lady who loves mango juice. Mary is a peddler who wants to sell as many mangoes as possible. This morning, out goes Jeanne in search of her favorite fruit. As she crosses a street corner, a soft voice breaks the silence in her mind.

Mary: My dear beautiful lady, would you come and check my mangoes out?
Jeanne: Ok. No problem. Indeed, I do need some mangoes to make juice.
Mary: That’s a good idea. Today they’re fresh and delicious.
Jeanne: How do you sell these Francique mangoes?
Mary: Forty cents each!

Narrator: Freeze! How much does Jeanne owe for the Francique mangoes?
Mary: Mmm! Let’s see... $__________.
Jeanne: Ok! I also need some Cinnamon mangoes.
Mary: That’s a good idea. Today they’re sweet and juicy.
Jeanne: Is it the same price?
Mary: Nope! These are two for a dollar.
Jeanne: Ok. No problem. I’m taking three.

Narrator: Freeze! How much does Jeanne owe for these Cinnamon mangoes?
Mary: Mmm! Let’s see... $__________

Narrator: Freeze! How much does Jeanne owe in all?
Mary: Mmm! Let’s see... $__________

Narrator: Freeze! How much change (if any) has Jeanne left for Mary? Explain.
Mary: Mmm! Let’s see... $__________
Mango Juice Sample Questions

1. Who is Mary? Why does she want the mangoes for?
2. Who is Jeanne? What’s her desire?
3. What is a peddler?
4. Who begins the conversation? Jeanne or Mary? Cite some textual evidence.
5. Which mango brand is more expensive? By how much?
6. What adjectives are used to determine the *Francique* mangoes?
7. What adjectives are used to describe the Cinnamon mangoes?
8. Why does Jeanne use the determinant “crispy” to characterize her 5-dollar bill?
9. How much change has Jeanne left for Mary? How do you know?
Two days following her birthday party, Marie returned into the neighborhood store where she had purchased the beverages. She wanted to redeem the bottles. Once there, she dashed to the machine and began inserting the empties. When she finished, she pressed a button, and a total of one dozen nickels and dimes chimed into a shiny metal tray at the bottom. The monetary mixture amounted to one dollar. Astonishingly, Marie realized that there were twice as many dimes as nickels.

What activities would you customize for your students and in what subject areas?
Sample Questions

- Who went to the neighborhood store?
- When did Marie go to the store?
- What did Marie do after her birthday?
- Where did Marie insert the empty bottles?
- How did Marie go to the store?
- Why did Marie return to the store?
- Name one synonym for “beverages.”
- What does the word “redeem” mean?
- What is the value of one dime?
• What does the word “mixture” signify?

• How do you interpret “twice as many?”

• What is the value of one nickel?

• What is “one dozen?”

• Why does the author use the perfect past tense “had purchased” instead of the simple past tense “purchased?”
My dad gave me a one-dollar bill
‘Cause I’m his smartest son,
And I swapped it for two shiny quarters
‘Cause two is more than one!
And then I took the quarters
And traded them to Lou
For three dimes__ I guess he don’t know
That three is more than two!
Just then, came old blind Bates
And just ‘cause he can’t see
He gave me four nickels for my three dimes,
And four is more than three!
And I took the nickels to Hiram Coombs
Down at the seed-feed store,
And the fool gave me five pennies for them,
And five is more than four!
And then I went and showed my dad,
And he got red in the cheeks
And closed his eyes and shook his head__
Too proud of me to speak!

Shel Silverstein, Where the Sidewalks Ends
1. Why did Dad give the author a one-dollar bill?
2. Was the author really smart? How do you know?
3. Why did the author exchange the dollar for two quarters?
4. Name one adjective that determines the quarters.
5. Why did the author go and trade with Lou, Bates, and Coombs?
6. What did the author think of Hiram Coombs?
7. Was the author right in his characterization of Coombs? Why?
8. Underline all adjectives in the poem.
9. In reality what did the father think of the author, his son? Why?
10. How did the authors interpret his father’s feelings at the end? Why?
11. Name all regular verbs in the poem. How do you know they are regular?
12. Name all irregular verbs in the poem. How do you know they are irregular?
13. List all pronouns in the poem and indicate which nouns they replace.
14. Write the poem in your own words using the present tense.
Blessed am I with some special qualities! Mathematicians would say unique properties. ‘Cause my two **diagonals** are **perpendicular**, Thus definitely anointing me in particular.

Above all, I am a **full-fledged** Polygon. From my **vertices**, multiple sides can be drawn. Moreover, I do inherit traits of Parallelogram, “Cause parallelism sprawls all over my diagram.

Any clone of mine **inevitably** is a Rectangle, “Cause each angle is 90 degrees, a **right angle**. But don’t be fooled! All rectangle copies Don’t necessarily possess my qualities.

To some extent, I do look like a **Rhomboid**. Also, I share a thing or two with a **Trapezoid**. Definitely, I am a **Rhombus**; my sides are the same. Oh, boy! Can’t you still guess my name.
Sample ELA/ENL Questions

1. Which specific polygon the poem is talking about? How do you know?

2. Underline the bolded words and explain their meanings.

3. What does “to some extent” mean?

4. Underline all adverbs in the text and provide their meanings.

5. Rewrite each stanza in (prose) your own words with a relevant illustration.

6. Formulate your own questions.
Learning Log

- Journal entries (on what has been learned) specified by the teachers

- Not all journal entries are learning logs.
Dr. Aida Walqui

QTEL
Planning Your Language Objectives

Language Form
- Sounds of Language
- Form of words (nouns, pronouns, verbs, adjectives, adverbs)
- Structure of sentences; sentence patterns
- Length, Complexity of Sentences
- Connectors; Cohesive ties

Language Functions
- Kinds of communicative tasks students must accomplish (name, tell, compare, describe, explain, apologize, insult, request)

Language Style
- Formal vs. informal/casual
- Social vs. academic
- Colloquial Language (idioms; common similes, metaphors)

Language Content & Use
- Semantics (vocabulary; phrasing; tone)
- Pragmatics (intended meanings; language as used in different contexts)
Suggested Language Objectives:

**Language Forms**
- Sounds of Language (*th* in *theorem*, *v* in *vertex*)
- Parts of Speech (*prepositions*, *commands*, *comparative/ superlative adjectives*)
- Sentence Patterns (*clauses*, *modals* (*could*, *may*), *possibility*—*if...then, given...*)
- Length, Complexity of Sentences

**Language Functions**
- Kinds of communicative tasks students must accomplish:
  - Draw and label,
  - find,
  - consider,
  - compare,
  - determine,
  - explain,
  - state, etc.
Common communication in the mathematics classrooms

Dr. Cloud

- discussing ideas and asking questions,
- summarizing instructional key ideas or defining a term
- following and giving instructions,
- recounting how a problem was solved,
- explaining thinking/reasoning aloud,
- giving reasons for a response,
- showing problem solving steps to display knowledge on tests
- describing procedures to follow
SKILLS

(Computational Fluency)

Speed + Accuracy
Be aware of and be sensitive to the differing techniques that our students may bring into the classroom.

It is quite possible that their exotic methods might be more efficient than ours.
How to perform each operation mentally and efficiently. Explain your process.

\[19 + 11 = \]
\[52 - 13 = \]
\[23 + 17 + 10 = \]
\[74 - 26 = \]
Mental Math

Multiplying by a power of 10

- 1
- 10
- 100
- 1,000
- 10,000
What is the most efficient way to get the results? Why?

16 \times 0.50 =
28 \times 0.50 =
36 \times 0.50 =
40 \times 0.50 =
What is the most efficient way to get the results? Why?

16 \times 0.25 = 

28 \times 0.25 = 

36 \times 0.25 = 

40 \times 0.25 =
What is the most efficient way to get the results? Why?

16 \times 0.75 = \\
28 \times 0.75 = \\
36 \times 75 = \\
40 \times 0.75 =
Asian Multiplication

12 \times 13 = 156
24 \times 35 = 840
$30 \times 245 = 7,350$
Metric System Prefixes

kilo = 1,000

hecto = 100

deca = 10

Standard = 1 (meter, gram, liter)

deci = .1 = \frac{1}{10}

centi = .01 = \frac{1}{100}

milli = .001 = \frac{1}{1000}
The System Metric Table for Mass/Weight

<table>
<thead>
<tr>
<th>kg</th>
<th>hg</th>
<th>dag</th>
<th>g</th>
<th>dg</th>
<th>cg</th>
<th>mg</th>
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</thead>
<tbody>
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<td>Thousands</td>
<td>Hundreds</td>
<td>Tens</td>
<td>Ones</td>
<td>Tenths</td>
<td>Hundredths</td>
<td>Thousandths</td>
</tr>
<tr>
<td>1,000</td>
<td>100</td>
<td>10</td>
<td>1</td>
<td>.1</td>
<td>.01</td>
<td>.001</td>
</tr>
</tbody>
</table>

kilogram hectogram decagram gram decigram centigram milligram
### The System Metric Table for Capacity

<table>
<thead>
<tr>
<th>kl</th>
<th>hl</th>
<th>dal</th>
<th>l</th>
<th>dl</th>
<th>cl</th>
<th>ml</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thousands</td>
<td>Hundreds</td>
<td>Tens</td>
<td>Ones</td>
<td>Tenths</td>
<td>Hundredths</td>
<td>Thousandths</td>
</tr>
<tr>
<td>1,000</td>
<td>100</td>
<td>10</td>
<td>1</td>
<td>.1</td>
<td>.01</td>
<td>.001</td>
</tr>
</tbody>
</table>

kiloliter  hectoliter  decaliter  liter  deciliter  centiliter  milliliter
### The System Metric Table for Length

<table>
<thead>
<tr>
<th>km (Thousands)</th>
<th>hm (Hundreds)</th>
<th>dam (Tens)</th>
<th>m (Ones)</th>
<th>dm (Tenths)</th>
<th>cm (Hundredths)</th>
<th>mm (Thousandths)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,000</td>
<td>100</td>
<td>10</td>
<td>1</td>
<td>0.1</td>
<td>0.01</td>
<td>0.001</td>
</tr>
</tbody>
</table>

kilometer, hectometer, decameter, meter, decimeter, centimeter, millimeter
The System Metric (Table for Length)

<table>
<thead>
<tr>
<th>km</th>
<th>hm</th>
<th>dam</th>
<th>m</th>
<th>dm</th>
<th>cm</th>
<th>mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thousands</td>
<td>Hundreds</td>
<td>Tens</td>
<td>Ones</td>
<td>Tenths</td>
<td>Hundredths</td>
<td>Thousandths</td>
</tr>
<tr>
<td>1,000</td>
<td>100</td>
<td>10</td>
<td>1</td>
<td>.1</td>
<td>.01</td>
<td>.001</td>
</tr>
<tr>
<td>1000</td>
<td>100</td>
<td>10</td>
<td>1</td>
<td>1/10</td>
<td>1/100</td>
<td>1/1000</td>
</tr>
<tr>
<td>1000</td>
<td>100</td>
<td>10</td>
<td>1</td>
<td>1/10</td>
<td>1/100</td>
<td>1/1000</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1/10</td>
<td>1/100</td>
<td>1/1000</td>
</tr>
</tbody>
</table>

Note: 1 km = 1,000 m

5 km = \[5,000\] m
### System Metric Table for Length

<table>
<thead>
<tr>
<th>km</th>
<th>hm</th>
<th>dam</th>
<th>m</th>
<th>dm</th>
<th>cm</th>
<th>mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thousands</td>
<td>Hundreds</td>
<td>Tens</td>
<td>Ones</td>
<td>Tenths</td>
<td>Hundredths</td>
<td>Thousandths</td>
</tr>
<tr>
<td>1000</td>
<td>100</td>
<td>10</td>
<td>1</td>
<td>.1</td>
<td>.01</td>
<td>.001</td>
</tr>
</tbody>
</table>

5 m = .005 km
### System Metric Table for Length

<table>
<thead>
<tr>
<th>km</th>
<th>hm</th>
<th>dam</th>
<th>m</th>
<th>dm</th>
<th>cm</th>
<th>mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thousands</td>
<td>Hundreds</td>
<td>Tens</td>
<td>Ones</td>
<td>Tenths</td>
<td>Hundredths</td>
<td>Thousandths</td>
</tr>
<tr>
<td>1000</td>
<td>100</td>
<td>10</td>
<td>1</td>
<td>.1</td>
<td>.01</td>
<td>.001</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>.6</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7465 m = 74.65 hm
### System Metric Table for Area

<table>
<thead>
<tr>
<th>km²</th>
<th>hm²</th>
<th>dam²</th>
<th>m²</th>
<th>dm²</th>
<th>cm²</th>
<th>mm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$5 \text{ km}^2 = 5,000,000 \text{ m}^2$

Note: $1 \text{ km}^2 = 1,000,000 \text{ m}^2$
## System Metric Table for Area

<table>
<thead>
<tr>
<th>km²</th>
<th>hm²</th>
<th>dam²</th>
<th>m²</th>
<th>dm²</th>
<th>cm²</th>
<th>mm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>05</td>
<td>00</td>
<td>00</td>
<td>00</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5,000,000 m² = 5 km²
<table>
<thead>
<tr>
<th>km²</th>
<th>hm²</th>
<th>dam²</th>
<th>m²</th>
<th>dm²</th>
<th>cm²</th>
<th>mm²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.5</td>
<td>86</td>
<td>70</td>
<td>60</td>
</tr>
</tbody>
</table>

5.867,060 m² = 58,670.60 cm²
System Metric Table for Volume/Capacity

<table>
<thead>
<tr>
<th>km³</th>
<th>hm³</th>
<th>dam³</th>
<th>m³</th>
<th>dm³</th>
<th>cm³</th>
<th>mm³</th>
</tr>
</thead>
<tbody>
<tr>
<td>005</td>
<td>000</td>
<td>000</td>
<td>000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5 km³ = 5,000,000,000 m³

Note: 1 km³ = 1,000,000,000 m³
# The Language of the Computer

## The Binary System

<table>
<thead>
<tr>
<th>$2^5$</th>
<th>$2^4$</th>
<th>$2^3$</th>
<th>$2^2$</th>
<th>$2^1$</th>
<th>$2^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>16</td>
<td>8</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

## The Decimal System

<table>
<thead>
<tr>
<th>$10^5$</th>
<th>$10^4$</th>
<th>$10^3$</th>
<th>$10^2$</th>
<th>$10^1$</th>
<th>$10^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100,000</td>
<td>10,000</td>
<td>1,000</td>
<td>100</td>
<td>10</td>
<td>1</td>
</tr>
</tbody>
</table>

0 = 0
1 = 1
2 = 10
3 = 11
4 = 100
5 = 101
6 = 110
7 = 111
8 = 1000
9 = 1001
10 = 1010
11 = 1011
12 = 1100
13 = 1101
14 = 1110
15 = 1111
16 = 10000
CONCEPTS

Deep Understanding
Explain how $2 \times 3$ and $3 \times 2$ are similar and different.

Use real-life situations or manipulatives to represent your ideas.
Area vs. Perimeter

Tessellate 16 tiles in various configurations, i.e., 2×8. Then determine which configuration has the largest area, and which configuration has the longest perimeter. Explain why.
The Meaning of Division

\[
3 \div \frac{1}{2} = \frac{3}{1} \times \frac{2}{1} = 6
\]

- What real-life situation would require the division above?
- What does this division entail?
Concrete Division for Profound Understanding

Answer: 6

Six halves (1/2) mango
\[
\frac{3}{\frac{1}{2}} = 3 \times \frac{2}{1} = \frac{3}{1} \times \frac{2}{1} = \frac{3 \times 2}{1} = \frac{6}{1} = 6
\]
\[\frac{3}{4} \div \frac{1}{8} = 6\]
WHOLE-CLASS ACTIVITY

Faces of the Prism
# Faces of the Prism

<table>
<thead>
<tr>
<th>Prism Face</th>
<th>Area cm²</th>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$770</td>
</tr>
<tr>
<td>Back</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Right</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Left</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bottom</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>?</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Observation:**

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Theories & Frameworks
21st Century Paradigm

- Standards-based Model
- Constructivist Model
- Discovery Model
- Inquiry-based Model
- Project-based Model
- Japanese Model (Neriage)
- Chinese Model (Bantu)
- Organic-way Model (under study)
Organic-way Math

1. Group Work
2. Whole-Class Discussion
3. Teacher Insights
4. Student Reflection
5. Solo Practice

Teacher Notes & Reflections
Anatomy of Problem Resolution

1. Fact Finding & Understanding of the Problem
   - Read for understanding of the story problem
   - Read for understanding of the math terminology
   - Draw a T-Chart to record the given facts and the question(s)

2. Planning & Attacking the Problem
   - Ball-park / guess-estimate / predict the solution
   - Decide on the most efficient strategy
   - Represent the problem with graphic organizer(s) or pictures / drama
   - Write the solution steps showing your logical reasoning
   - Ask yourself whether the answer makes sense
   - Double-check the answer

3. Exploring & Securing the Solution
   - Expand the solution / explore other possibilities
   - Make connections and categorize the problem
   - Evaluate and appreciate the problem
Shawna has $1.60 in quarters, dimes, and nickels. There is an even number of quarters. There are more dimes than nickels. There are as many quarters and nickels combined as there are dimes. Find the number of each kind of coin. Explain. Show your work.

(Adapted from NCTM, November 2004 Issue)
Scaffolding Manipulatives Continuum

From most Concrete to most Abstract

realia (real things)
replica fake rubber plastic clay wood
Image picture photo
drawing sketch symbol graphic
gesture sign language
oral verbal
text print

MOST COMPREHENSIBLE INPUT

LEAST COMPREHENSIBLE INPUT

In quest for $i + 1$ (Krashen)
Robert Marzano (2005) Effective Instructional Strategies

1. Identify similarities & differences
2. Summarize & take notes
3. Reinforce efforts & make room for recognition
4. Provide homework & practice
5. Use also nonlinguistic representations
6. Structure cooperative learning
7. Set objective & provide feedback
8. Generate & set hypothesis
9. Provide cues (e.g., scaffolds and probing questions)
Scaffolding Strategies & Frameworks

1. Verbal-Visual-Word Association (VVWA)
2. Concept Map
3. Sentence Starters / prompts
4. Word Etymology & Vocabulary
5. KWL Chart
6. Anticipatory Guide
7. Frayer Model
8. Math Glossary Puzzle
9. Flow Chart / Table / Diagram
10. Math Poetry
11. Close Reading
12. Socratic Method for Teaching & Learning
13. Project-based Learning
14. Organic-way Math (under study)
15. Math Games (e.g., ORGABEZ)
16. Etc.
Task: Sort out beans only of four different colors. Count them, and complete the accompanying table, taking into account that each bean is worth $5.
Bermuda Fish Table

<table>
<thead>
<tr>
<th>Color</th>
<th>Quantity</th>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
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<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Total

Observation: I am surprised to see that
The Bermuda Fish

Total weight (whole fish) = 40 lbs.

Tail = 8 lbs.

Head = 10 lbs.

$25

Middle = ?
### Bermuda Fish Table

<table>
<thead>
<tr>
<th>Parts of Fish</th>
<th>Weight</th>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$25</td>
</tr>
<tr>
<td>Middle</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tail</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Whole Fish</td>
<td>40 lbs.</td>
<td></td>
<td></td>
<td></td>
<td>$64</td>
</tr>
</tbody>
</table>

Observation: I am surprised to see that
Familiar Surfaces

- Select three different two-dimensional objects (e.g., paper, door, book, table, and board).
- Measure them and determine their areas.
- Complete the table and construct a pie chart.

Note: Each square unit = $50
## Familiar Surfaces Table

<table>
<thead>
<tr>
<th></th>
<th>Area</th>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Object 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Object 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Object 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All Objects</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Area</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Observation:**

It is interesting to observe that ...
The Dream House

See folders of past PDs on desk
<table>
<thead>
<tr>
<th>Room</th>
<th>Floor Area</th>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Patio</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$200</td>
</tr>
<tr>
<td>Bedroom</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Living Room</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bathroom</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kitchen</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Whole House</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
New Library Proposal

Diagram showing the layout of the library with sections for Social Studies, English, Art, Info, Math, and Science.
<table>
<thead>
<tr>
<th>Room</th>
<th>Area</th>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social Studies</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$2,000</td>
</tr>
<tr>
<td>Science</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>English</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Math</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Info</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Art</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Entire Library</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1. A(0,5); B(4,5); C(4,1); D(0,1) 
   Area of ABCD?

2. E(8,1) 
   Area of BCE?

3. Area and perimeter of ABED?


5. Write your observation
<table>
<thead>
<tr>
<th>Polygon</th>
<th>Area</th>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABCD</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$32</td>
</tr>
<tr>
<td>BCE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ABED</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Observation: 

----------------------------------------------------------------------------------------------------------------------------
Pattern Blocks

1. Use a cm ruler to measure the dimensions of each polygon of the pattern-block set.

2. Find the area of each polygon, then complete the table, taking the hexagon as sum of the areas or the whole.

3. Write your observation of the table.
## Pattern-Block Area Table

<table>
<thead>
<tr>
<th>Polygon</th>
<th>Area</th>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parallelogram</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trapezoid</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hexagon (Whole)</td>
<td></td>
<td></td>
<td></td>
<td>$64</td>
<td></td>
</tr>
</tbody>
</table>

**Observation:**

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---
References

Wikipedia
Nancy Cloud
Aida Walqui
Organic-way Mathematics
Archangelo Joseph
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New York, NY 10003

Thank You