Examining Mathematical Knowledge for Teaching in Secondary and Post-Secondary Contexts

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Issues we’ve been thinking about

• An elementary teacher understanding multiple age-appropriate definitions of even.

• But what about a secondary teacher understanding multiple definitions of function?
Some things we have learned from research on teaching

• Content knowledge is not all that matters
  – Teachers’ having more courses in content is not strongly correlated with higher achievement for their students (Begle, 1979; Monk, 1994)

“The conclusions of the few studies in this area are especially provocative because they undermine the certainty often expressed about the strong link between college study of a subject matter and teacher quality.” (Wilson, Floden, & Ferrini-Mundy, 2002, p. 191)
Even more things we know from research on teaching

• Specialized content knowledge matter (aka “mathematical knowledge for teaching”)
  – used to do the “mathematical work” of teaching
  – to follow and understand students’ mathematical thinking
  – to evaluate the validity of student-generated strategies
  – shown to play a role in teachers’ practices and correlate with students’ learning (Ball & Bass, 2000; Hill et al 2004, 2005; Ma, 1999)
But…

• What does this mean for secondary and other non-elementary teachers?
• Important for practical reasons
  – Design of math programs for teachers
  – Capstone courses
• Important for theory-development reasons
  – How generalizable are the constructs?
Our goals for today

• Compare and contrast these issues at the elementary and secondary/undergraduate levels

• Spark conversation about these issues
Our plan

• Introduction to the issues
• Overview of two (provocative?) examples
• Small group discussion
• Concluding thoughts and discussion of future directions
Terms used in the community

- Content knowledge
- Pedagogical knowledge
- Pedagogical content knowledge
- Mathematical knowledge for teaching
Mathematical knowledge for teaching includes:

- Common content knowledge
- Specialized content knowledge
- Horizon content knowledge
Common content knowledge

- mathematical “knowledge of a kind used in a wide variety of settings – in other words not unique to teaching”; these are not specialized understandings but are questions that typically would be answerable by others who know mathematics” (Ball, Hoover Thames, & Phelps, p. 399)
Specialized content knowledge

• “the mathematical knowledge ‘entailed by teaching’ – in other words, mathematical knowledge needed to perform the recurrent tasks of teaching mathematics to students” (Ball, Hoover Thames, & Phelps, p. 399)
Horizontal content knowledge

• “an awareness of how mathematical topics are related over the span of mathematics included in the curriculum” (Ball, Hoover Thames, & Phelps, p. 403).
Karen’s example: Secondary

A teacher poses the following problem:
Suppose that a staircase comprises ten steps and that you can climb the stairs one or two steps at a time. In how many different ways can you climb these ten steps? (Rubel & Zolkower, 2007/2008).

The teacher has an image of one solution, using combinations and counting methods.
A group comes up with the following solution:
And they conclude:

- The pattern is the Fibonacci sequence. So for 10 steps there are 89 ways.
Issues to consider:

• Is this sequence really the Fibonacci sequence? Why?
• How does this solution connect to the combinatorial solution? Is this an important mathematical connection to make?
• Do you consider the knowledge needed to deal with this solution common content knowledge, specialized content knowledge, pedagogical content knowledge, or something else? Why?
Natasha’s example:

- “Specialized because it is not needed or used in settings other than mathematics teaching” (p. 396, Ball et al, 2008)
  - Consider the pedagogical entailments of particular examples
- But, mathematicians do things in their work that seem the same as what is needed for teaching:
  - Examine/check the validity/accuracy of problem solutions
  - Formulate responses about the validity/accuracy of the problem solutions
Natasha’s example, cont.:

• Elementary and secondary teachers generally do not examine the mathematical work of their peers.
• So, does that mean that the knowledge used by mathematicians while checking validity of student-generated solutions is common content knowledge for mathematicians?
Discuss with your neighbors

Karen’s example:
• Is this sequence really the Fibonacci sequence? Why?
• How does this solution connect to the combinatorial solution? Is this an important mathematical connection to make?
• Do you consider the knowledge needed to deal with this solution common content knowledge, specialized content knowledge, pedagogical content knowledge, or something else? Why?

Natasha’s example:
• Is the nature of the work that a mathematician does while looking at the work of a peer the same as examining the validity of the work of a student?
• Is the work the same when formulating a response to these different people?
• Is the knowledge used to do this work the same in both cases?
Summary comments

- Not clear how the definitions of CCK and SCK translate to non-elementary level contexts
- Learn from what has been done at elementary level and design our research in light of what we have learned
- More work needs to be done.
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References