Topic Proposal

THE UNEQUAL TUNING SYSTEM OF WELL-TEMPERAMENT AND ITS INFLUENCE ON KEY CHARACTERISTICS IN J.S. BACH’S WELL-TEMPERED CLAVIER

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The Unequal Tuning System of Well-Temperament and Its Influence on Key Characteristics in J.S. Bach’s Well-Tempered Clavier

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Abstract: The main focus of this thesis is to explore the integral role that the well-tempered tuning system—a system comprised of unequal sized semitones promoting systematic, smooth color change through modulation largely due to the variance in the size of the interval of the third—had on Bach’s compositional process in the Well-Tempered Clavier, and how temperament is essentially encoded in the music itself. By approaching theoretical analysis through the lens of temperament and analyzing for the prevalence, usage, and placement of specific types of intervals, and looking at the deeper implications on the structure of the piece based on both their melodic (interval size and linear pull) and harmonic (type of resonance they promote in terms of their placement in the overtone series) functions, I believe that we can come to a deeper understanding as to why certain subjects and motifs are used for specific keys, as well as why certain modulations and contrapuntal devices (doublings, treatment of dissonance, prevalence of dissonance and the manner in which it is resolved), make musical sense.

Theoretical score analysis will be broken down into two components: global statistical analysis concerned with characterizing the distribution of fifths and minor thirds across the entire work, and a detailed, comparative analysis of select minor mode prelude and fugues. The purpose of the former is to characterize correlations between prevalence of intervals and key area, suggesting compositional strategy to be a function of temperament, and purpose of the latter is to address the question of significance. To expound: the latter seeks to demonstrate how temperament controls the balance between dissonance and consonance—which respectively governs the flow of movement and stasis—and how altering temperament will in effect throw off this delicate balance and obfuscate the intended affect and musical integrity of a given piece.

The central goal of this research project is to examine the role and importance that musical temperament has had in the creation, as well as the musical integrity, of J.S. Bach's Well-Tempered Clavier—specifically, the objective is to demonstrate that the tuning system that Bach was working with was not only an determining factor in shaping his compositional process given a certain key, but more importantly an integral, indispensable element of the composition's overall musical affectation upon sonic realization.

The main claim that I wish to test in this dissertation is that Bach’s compositional process was distinctly a function of temperament. In other words, Bach was writing the Well-Tempered Clavier with a well-tempered turning system in mind, as opposed to a system that more resembled equal-tempering, and furthermore that this factor of tuning manifests itself to the listener through producing tensions and harmonies that are unique to that particular system. Essentially, the claim is that the tuning system—which governs the sizes of intervals—exerts direct control over the affect and musical integrity of each individual piece in the collective work, and that this was Bach’s express intent. To remove it and use a different tuning system in realization of the piece would not only effectively eradicate a dimension of the music, but also in fact alter the very aesthetic makeup and integrity of the piece as a whole, a process akin to the alteration of the hues of colors in a painting.

1 RESEARCH QUESTIONS AND OBJECTIVES
The ideas enumerated in the above paragraph are very broad; to realistically explore the validity of these statements relating to Bach’s compositional process and its relation to affect and listener experience would involve a leviathan project that encompasses the various modalities of historical inquiry, perceptual psychophysics, and theoretical analysis. While any inquiry of Bach’s compositional process as a function of temperament will inevitably incorporate all these elements, for this project I wish to focus my on a more theoretical, using score analysis as the main platform of study.

Essentially, the three research questions that I wish to address through my analysis are as follows:

1. Is there reasonable internal evidence (i.e. evidence that can be deduced from theoretical score analysis alone) to believe that Bach’s compositional process was indeed a function of well-temperament?
2. If (1) is affirmed, is temperament truly an integral factor to the overall musical integrity of a specific piece—or is the aesthetic and affectual implication essentially unaltered by temperament?
3. If (2) is affirmed, how is this specifically demonstrated through Bach’s compositional choices (i.e. what are the correlations between key, interval choice and usage, and treatment of counterpoint), and, specifically, how is the structure of a piece inherently altered through using the same interval class generated by equal temperament?

The purpose of addressing each of these questions—and indeed the central purpose of my study—is to underscore the importance of considering temperament when discussing the underlying musical structure within the pieces of Bach’s Well-Tempered Clavier, and to illuminate what is gained in terms of structural understanding and affect when one analyzes and performs the work under the lens of well-temperament, and conversely what is lost when a system of equal temperament is employed.

Before I provide a more detailed and comprehensive outline for the research method that I will employ for this dissertation, I think it is important to give a brief overview of the history of tuning and temperament in order to put things into context. Furthermore, this historical overview is not so much for didactic purposes as it is to demonstrate evidence supporting a reasonable faith in the truth-value of some critical premises in which I will construct the main argument of my dissertation upon.

2 HISTORICAL OVERVIEW

Prior to raising any discussion about Bach’s *Well-Tempered Clavier*, or even the more general topic of the historical development of tuning systems, it is crucial to first establish the importance of tuning and temperament and the integral role they have to play in our aesthetic experience of music. This role is not just practical; in other words, tuning is not just a prerequisite condition to be satisfied before the compositional and performance processes, but rather is of tantamount importance to any other compositional device employed that should contribute to a piece’s aesthetic and emotional import. This is due to the nature of tuning, in that it essentially governs interval creation, which is the foundation of the building blocks of harmony. Thus, tuning is not only a scientific endeavor, but more
importantly also an artistic one. Consider: there are twelve different nominal simple
intervals (not exceeding the octave) that exist in traditional Western tonal practice, counting
the octave. Under the tuning system of equal temperament, there are indeed only twelve
different types of intervals (there is only one size major third at 400, one perfect fifth at 700
cents, etc.) However, under a system of unequal, well-temperament such as Werckmeister
III, there are a total of 39 different types of intervals (four different sizes of semitones, four
different major thirds, two different sizes of fifths, etc.)—more than threefold of an
increase.

It is commonly accepted among musicians that different types of intervals have the power
to elicit different affectual responses; historically, this was extended to encompass different
qualities and sizes of the same interval. Prior to the late 19th century, tuning by ear largely
meant playing an interval melodically, and listening to the quality of the interval instead of
playing an interval harmonically and listening to beats. During the development of equal-
beating meantone temperament in the mid-17th century, tuners began to listen to the beating
of fifths, but thirds, which they thought were too narrow an interval to be able to reliably
tune through beating, were still tuned melodically, listening to the quality. The quality, or
color, of intervals was of immediate importance to tuners, musicians, and theorists, who
would discuss and compare different sizes of thirds and their corresponding differences in
affect (e.g. the calmness induced by a pure, 5:4 ratio major third as opposed to the frenetic,
excitable effect created by a wide, 81:64 ratio Pythagorean third).

At this point, the question inevitably arises: how did all these different sizes of one type of
interval come into use? It does not make much immediate sense that, given the pure,
simple-ratio intervals that arise naturally from the harmonic series, why music would not
have developed to just utilize those intervals that are most harmonious and found readily in
the natural world. The shorthand answer to this is that, this indeed was the way in which
people first went about discretizing the pitch continuum, but quickly ran into a problem if
they wanted their intervals to possess the quality of invariance across transposition.

2.1 The Discrepancy of the Comma and the Need for Temperament

If we look at the harmonic series, we can see a close relationship between the first two
intervals (omitting the octave) that are found in the series—the fifth and the major third—
and the way harmony and chords are constructed in Western tonal practice. The roots and
evolution of the Western tonality, as well as the diatonic scale itself, lie directly in the
physics of acoustics. The way that pitches of the diatonic scale have been selected over the
centuries is directly related to the physics of sound and the overtone series, and our
perceptual system’s favoring of simple ratios. The earliest tuning system—just intonation—
took its notes directly from the simple ratio intervals found in the harmonic series. The
reason behind this was that these simple ratio intervals allowed for a complete integration
of sound devoid of beating—a perfect, harmonious blend. However, the problem of just
intonation was that, if one is to preserve the purity of all intervals, that is, to use only just
intervals in the construction of the diatonic scale of one particular key, these same intervals
in another key would be hopelessly out of tune. In other words, perfect, simultaneously,
harmoniousness across all keys is a mathematical impossibility. The illustration detailed in
the following paragraph will attempt to shed more light on this concept:
The fifth—the first interval found at the third partial in the harmonic series—enjoys an especially privileged role in Western tonal music. Essentially, Western tonality is a system that is built up on fifths. Because of the importance of this interval, musicians and theorists quickly realized the importance that these fifths be invariant and uniform across different starting pitches. One of the earliest forms of just tuning, Pythagorean tuning, strove to preserve the purity of these fifths as paramount. In Pythagorean tuning, the chromatic scale was constructed by stacking pure fifths, expanding on both ends until the circle was completed. However, the problem was that when the circle was complete (i.e. if one were to start on C, and after a series of fifths, arrive back at C), the final pitch was not the same exact pitch as the initial one, but rather sharper—a difference of about 23.5 cents, known as the Pythagorean comma. In order to preserve the justness of the octave, the last fifth in Pythagorean temperament (traditionally G♯ – E-flat) was not truly a fifth but a wolf diminished sixth that was too narrow for the ear to reasonably interpret as a fifth. This interval was completely out of tune, rendering any key that utilized it subsequently unusable.

It was precisely this tradeoff between purity of intervals and key accessibility, caused by the mathematical phenomenon of the comma, that necessitated the art of tempering in search of a balance between the two interests.

2.2 Meantone Temperament, the Development of Well-Temperament and the Characteristics of Keys

Sometime in the late 15th century there was a shift between favoring the sound of the major third over the fifth, and with this came the development of meantone temperament. The more precise definition of meantone has to do with the placement of a whole tone at exactly the mid point between its flanking neighbors, but more practically, meantone came to be known as a temperament that sought to temper fifths in equal amounts (not counting the wolf diminished sixth) in order to achieve pure major thirds.

In order to achieve equal fifths, tuners had historically employed a system called equal-beating, in which they would ensure that each fifth beat at the same amount, and believed that this method would mathematically yield fifths of the same size. This was, in fact, mathematically incorrect—in order to obtain equal size fifths at different starting pitches, beating rate must vary according to the frequency of the starting pitch. However, this mathematical principle was not known to scientists and tuners back during the development and widespread use of meantone, so interestingly, instead of achieving equal sized fifths, fifths varied in size, and in a gradual and systematic way. As a result, the major third also varied in size, with the major third of the tonic triad for F Major, C Major, and G Major being closest to pure, and gradually increasing in width as one moved away from these keys. This system also supported the characteristic of keys, as the major third of the tonic triad of different keys were of different sizes. However, while meantone temperament was favorable in that it preserved the purity of diatonic keys and supported key characteristics, modulation was still limited. In meantone temperament, the intervals of C♯-F, F♯-B-flat, and G-sharp-C, and B-E-flat, were wolf diminished fourths, about 13 cents wider than Pythagorean thirds, far too wide for the ear to be able to accept as in tune. As a result, a third of the keys were still unusable, and modulation to certain chords or key areas was restricted.
The system of well-temperament was first recorded, and expounded upon in detail, by Andreas Werckmeister in his treatise *Musikalische Temperatur*, in 1691. It was a system that was essentially the logical completion of meantone temperament, and historically, it is more accurate to classify well-temperament as modified meantone temperament, as it was a compromised version of meantone that allowed for free modulation across all keys. Well-temperament still preserved the purist thirds for the diatonic keys, and had the size of these thirds gradually increase as one modulates further away. However, what well-temperament did was that it got rid of the wolf-diminished sixth, thus making the remote keys of B, C#, F#, and A-flat usable, as the thirds that were previously wolf diminished fourths in meantone temperament were now Pythagorean thirds—wider and completely different in sound than just thirds, but acceptable to the ear.

3 ANALYSIS

Having established a proper historical framework, I can now fittingly proceed into a more detailed discussion of analysis; this section will essentially provide the reasoning behind and framework to the two types of analyses that will comprise my project, and outline the respective methods of each.

The score analysis proposed in section one can be broken down into two segments: global statistical analysis that looks at the distribution of the rate of occurrence and prominence of certain intervals—primarily the minor third and perfect fifth (for the purpose of this study I will refrain from analysis of the major third, which usage and distribution has already been examined in previous studies (Barnes 1979)). I will be looking for both horizontal and vertical instances of these intervals, across the entire work, and local, detailed analysis that looks in depth at a select number of pieces in the work, still focusing on intervals but looking at more specifically at how counterpoint is treated, and how these intervals are approached and resolved in order to examine how temperament is used as a coloration device in terms of creating musical structure from the interplay between dissonance and consonance. For this second, fine-grained analysis portion of my project, I wish to specifically focus on the Pythagorean minor preludes and fugues (b-flat, f, d-sharp), and how they compare to that of the "just" 6:5 ratio minor third prelude and fugues (a, d, e).

The following is an outline of the logic behind why I have chosen these two types of analysis to conduct for my project—as well as why I have chosen to specifically look at the Pythagorean and just minor pieces—with a statement of purpose delineated at the conclusion of the section.

Premises

1) The Pythagorean minor keys (along with their relative Pythagorean major keys) are the keys that were in a sense "unlocked" through the system of well-temperament, in that these were the meantone wolf keys that were previously inaccessible. These keys differ from the traditional meantone diatonic keys in that they contain the intervals of the pure fifth (as opposed to narrower, tempered fifths) and the Pythagorean third (a narrower minor third—the semiditone—for minor keys, and wider major
third for major keys), as opposed to meantone derived thirds that are close to just.

2) At a preliminary glance, Bach’s treatment of these Pythagorean minor preludes and fugues in Book I differ from those of other keys in that a) he employs a stile antico approach to these compositions, in that the constructs of the fugal subjects and episodic material are distinctly modal in nature, and the contrapuntal treatment of dissonances are more conservative; b) within the framework of his prima pratica treatment of counterpoint, he often features and draws attention to seconda pratica type dissonances (minor 9th leap in the b-flat minor fugue, augmented 4th leap in the f-minor fugue) that are not found in the preludes and fugues from the more diatonic keys.

3) We have reasonable evidence to believe that Bach's treatment and choice of certain intervals for employment within a certain key was very much a function of the temperament that he was working with. One viable means of demonstrating this notion in a more concrete fashion is through statistical analysis, showing that the distribution of intervals (the major third, for example) is not uniform across all starting pitches, but proportionally skewed with high values corresponding to the diatonic keys that fall at a steady rate as one progresses towards the Pythagorean keys (Barnes, 1979). It is also evident that temperament, which governs the purity of certain intervals at the compromise of others, has historically exerted an influence over the rules of counterpoint of a specific era.

Corollary (of premise 3):

One of the principles behind selection—or avoidance—of certain types of intervals is the very basic notion of the preservation of consonance in music, and one can regard Barnes’ aforementioned study, as well as the historical predilection towards certain contrapuntal gestures that draw attention to specific intervals, as a testament to this principle of maximizing consonance. To wax slightly philosophical for a moment though, the ultimate effect and import behind consonance cannot be appraised without the presence of dissonance; in essence, the notion of consonance is a perceptual one, and it is governed by its relationship with dissonance, which determines the way these consonances are presented and ultimately received. Thus, the maximization of consonance depends not only on how much raw consonance (i.e. presence of pure over Pythagorean or tempered consonant intervals) is present in the system, but also how these consonances relate to the dissonances present. Practically, this can manifest itself in two different ways: a) the strategic placement of dissonance in a system can make the advent of a pure consonant interval even more powerful, and b) non-pure consonant intervals can be perceived as more consonant through preceding dissonance, an exercise in the power of relativity, if you will. Furthermore, it is important to consider that at times, consonance may not be the ultimate function that we wish to maximize if a certain musical affect is to be achieved; rather, perhaps more germane a matter to be discussed is the
relationship between the dissonance and consonance in a given system, and the organization of how each is approached and resolved.

Given these premises, I want to focus my project on the following two types of analyses:

1) Global statistical analysis of the prevalence of fifths and minor thirds, weighted by their respective function (harmonic/melodic/passing) in and for each specific key area, similar to the analysis performed on major thirds by Barnes to see if these distributions are non-uniform, and if so, whether or not there is a correlation between prevalence of occurrence and key. A positive correlation between the frequency of occurrence of these specific intervals and key could imply that Bach's compositional process was directly motivated and influenced by temperament.

2) A comparative study between the diatonic minor (a, e, and d) and Pythagorean minor (b-flat, f, and d-sharp) preludes and fugues, with focus on how Bach deals with the usages and resolution of dissonances, as well as his treatment of imperfect and perfect consonances, especially within the subject and at cadential points. My primary mode of analysis here will be intervallic analysis: on the horizontal domain, looking at how either consonant and dissonant intervals are featured as salient through frequency of occurrence, weighted by their prominence through looking at their placement in terms of underlying rhythmic and metric structures, and on the vertical domain, looking at how dissonances are resolved and consonances approached, as well as the types of cadences utilized, and which intervals are favored and brought to the forefront through voicing and doubling.

What I want to demonstrate through these analyses is that Bach's compositional process was indeed a function of well-temperament, and that furthermore, distinct from the idea behind meantone temperament, his treatment of counterpoint and employment of certain intervals over others was not so much motivated by the principle of avoidance but rather more about the purposeful usage of these distinct intervals to allow him access to a certain type of affect that is not as readily attainable through other types of the same interval. This is not to discount the validity of and importance of examining the issue of temperament using the principle of avoidance, which was the ideological framework that Barnes chose to adopt in his 1979 study examining the prominence and distribution of major thirds. However, Barnes’ inquiry warranted such an approach because he was largely concerned with determining the specific type of temperament that Bach was using. This dissertation instead is focused on how intervals and temperament are used as a expressive device, and the reason behind looking at the specific usage and resolution of the interval of the pure fifth and the Pythagorean minor third is that these intervals are, in many regards, intervals reminiscent of an older musical tradition that were essentially removed from the vocabulary with the widespread acceptance of meantone pure thirds and narrow fifths. The fact that Bach specifically chooses a stile antico approach to these Pythagorean minor compositions leads me to believe that there is a salient relationship between temperament and compositional style, in that a certain compositional style may serve to bring to the forefront the prominence of certain intervals.
At the heart of this relationship between temperament and compositional style is the interaction between dissonance and consonance—and herein lies the importance of adherence towards the original temperament in which a compositional was conceived under; to represent the composition with a different type of temperament would inevitably bring about an alteration of the intervals employed, which would in turn disrupt the balance between dissonance and consonance that is so integral to the affect that a particular piece carries.

3.1 Sample Study: D# Minor Fugue, BWV 853 (a3)

In this section is appended a skeletal sample of the method of intervallic analysis that I will use to examine the usage and treatment of melodic intervals in fugal subject statements, looking at their overall distribution, frequency of occurrence, and their interaction with other voices on the vertical axis. The eventual goal of such analyses is a demonstration of the importance of the given interval and its central role to the piece in evoking a particular effect, and furthermore, how the interval’s integrity is inextricably tied to temperament.

The following table is a subject entry intervallic analysis on the d# minor fugue from Book I (BWV 853), one of the Pythagorean minor fugues, focusing on the melodic and harmonic treatment of the interval of the fifth. As seen through the analysis, the interval of the fifth (and melodic fourth) are very prominently featured and emphasized; in melodic form it comprises the opening of the subject statement and closing gesture of the fugue, as well as is presented in harmonic form through placing modified statements of the subject, having the opening gesture outline a fourth, in stretto. An illustration of this can be seen in the beginning of the stretto recto and inverso exposition (m. 52): the ascending fourth gesture of a#-d# is iterated in each voice, presented sequentially in stretto staggered by a quarter note, yielding a sequence of knelling vertical perfect fifths d#-a#. The previous exposition (stretto inverso, m.44) also began with knelling fifths a#-d# (melodic descending), also derived through modification of the inverted subject to contain two iterations of the interval of a fifth, instead of a fifth and a fourth.

Through examination of Bach’s usage of the fifth throughout the piece, it is evident that he is exploiting the fifth’s acoustical resonant property to create a sense of space and solemnity, and it is important to note that, in Werckmeister III, 6 out of 7 of the types of fifths that are used in the subject statements of the piece are pure intervals, which would maximize this resonant effect. B-F# is the only narrow fifth that is featured, and is used only as a melodic fourth in the piece. Additionally, it is never used as the opening interval of the subject statement, except for one iteration in mm.24-25 in which its primary function is to create the flat-II dissonance against the other voices to imply Phrygian harmony.

**Key Schema (and Cadential Points):** i-III-v-III-iv-i-iv-III-i-III-VI-i

**Subject Entries:**
Total entries: 36
- d#-a# outlined (fifth or fourth) (pure): 24
- g#-d# (pure): 13
- a#-e# (pure): 8
- c#-g# (pure): 8
### Summary

- **f#-c# (pure): 6**
- **b-f# (narrow): 5**
- **e-b (pure): 1**

<table>
<thead>
<tr>
<th>Bar</th>
<th>Exposition</th>
<th>Key Scheme/Cadence</th>
<th>Measure</th>
<th>Transposition Level</th>
<th>Fourth and Fifth Outlined</th>
<th>Key Area</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-19</td>
<td>Recto</td>
<td>i-III-v; implied cadence on III, PAC on v</td>
<td>1-3</td>
<td>0</td>
<td>5: d#-a# 4: d#-g#</td>
<td>i (d# minor)</td>
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<td></td>
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<td></td>
<td>3(b.3)-5</td>
<td>7 (fifth) – tonal answer</td>
<td>4: a#-d# 4: a#-d#</td>
<td>v (a# minor)</td>
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<td>8-10</td>
<td>0</td>
<td>5: d#-a# 4: d#-g#</td>
<td>i</td>
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<td></td>
<td>12-15</td>
<td>7-tonal answer</td>
<td>4: a#-d# 4: a#-d#</td>
<td>i</td>
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<td>19-30</td>
<td>Stretto</td>
<td>v-III; Phrygian half cadence in key area v</td>
<td>19(b.3)-22</td>
<td>7 (fifth) – subject</td>
<td>5: a#-e# 4: a#-d#</td>
<td>v (a# minor)</td>
<td>Slight modification in rhythm (extra quarter rest between leap of the fourth interval)</td>
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<td></td>
<td>recto</td>
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<td>20-22</td>
<td>7 –subject</td>
<td>5: a#-e# 4: a#-d#</td>
<td>v (a# minor)</td>
<td>Canon at the octave, stretto at the half bar</td>
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<td></td>
<td>(canon;</td>
<td></td>
<td>24-26</td>
<td>2 (major second) – tonal answer/modifi ed subject</td>
<td>4: e#-a# 4: d#-g#</td>
<td>e# phryian</td>
<td>Answer begins on e# and returns back to e# approached by minor second descent (f-e#, bar 26). Even though this is not the technical conclusion of the answer in terms of correct number of notes given, this can be an implied ending based on a) the answer began on e#, and given the melodic schema of the original answer, we expect it to also end on e# as well, b) the presence of a half note leads the ear to halt on this note and c) a subsequent statement of the subject is presented right afterwards in elision</td>
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<td>canon-like</td>
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<td>tonal answer</td>
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<td>descending sequence)</td>
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<td>24(b.2)-27</td>
<td>7 (fifth) – tonal answer/modifi ed subject</td>
<td>4: a#-d# 4: g#-c#</td>
<td>F# lydian</td>
<td>Stretto at the quarter bar, C# major could also be implied, but F# Lydian is more compelling as descending sequence suggests motion leading towards F#</td>
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<td>24(b.3)-25</td>
<td>3 (minor third) – tonal answer/modifi ed subject, incomplete statement</td>
<td>4: f#-b</td>
<td>VI (B major, or IV/III), Phrygian harmonies implied against</td>
<td>On one hand, this answer, in isolation, implied the key area of VI (or IV/III, possibly setting up the modulation to key area of III that happens in bar 30), but on the other hand, in a</td>
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<td>the top two voices</td>
<td>more general context draws attention to the minor second above the tonic, implying Phrygian mode. This is so because the main section is in v (a# minor), V/v (e# was just implied), and the two preceding voices started on e# and a# respectively. Thus, the statement of f# and b in this voice almost serves as drawing attention to the minor second interval above the tonic note, a hallmark of the Phrygian mode.</td>
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<td>26(b.4) -29</td>
<td>0 – tonal answer/modified subject</td>
<td>4: d#-g# 4: c#-f#</td>
<td>V/III (C# major)</td>
<td>Again, the descending minor second that marks the end of the answer statement gives this iteration may suggest a Phrygian flavor</td>
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<td>27(b.2) -30</td>
<td>5 (fourth) – tonal answer/modified subject</td>
<td>4: g#-c# 4: f#-b</td>
<td>III (F# major)</td>
<td>Ends section in PAC</td>
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<tr>
<td>30-42</td>
<td>Inverso</td>
<td>III-iv-i; first statement of inverso in F# major, cadence in g# minor, subsequent two subjects given in e minor</td>
<td>30-33</td>
<td>3 (minor third)</td>
<td>4i: f#-c# 5i: g#/c#</td>
<td>III (F# major)</td>
<td>Inverted subject (answer)</td>
</tr>
<tr>
<td>36-38</td>
<td></td>
<td></td>
<td>7 (fifth)</td>
<td>4i: g#-d# 5i: a#/d#</td>
<td>i</td>
<td></td>
<td></td>
</tr>
<tr>
<td>39-41</td>
<td></td>
<td></td>
<td>0</td>
<td>4i: d#-a# 4i: d#-a#</td>
<td>i</td>
<td></td>
<td></td>
</tr>
<tr>
<td>44(b.3) -47</td>
<td></td>
<td></td>
<td>7 (fifth)</td>
<td>5i: a#-d# 5i: a#-d#</td>
<td>i</td>
<td>Canon at the octave (modified inverted subject with two statements of the interval of the fifth)</td>
<td></td>
</tr>
<tr>
<td>45-47</td>
<td></td>
<td></td>
<td>7 (fifth)</td>
<td>5i: a#-d# 5i: a#-d#</td>
<td>i</td>
<td>Modified inverted subject (two statements of the fifth)</td>
<td></td>
</tr>
<tr>
<td>47(b.3) -50</td>
<td></td>
<td></td>
<td>0</td>
<td>5i: d#-g# 4i: d#-a#</td>
<td>iv</td>
<td>Inverted subject, not modified</td>
<td></td>
</tr>
<tr>
<td>47(b.4) -</td>
<td></td>
<td></td>
<td>5 (fourth) – tonal answer</td>
<td>4i: g#-d# 5i: a#-d#</td>
<td>i</td>
<td>Incomplete statement of inverted answer; semi-augmented</td>
<td></td>
</tr>
<tr>
<td>52-61</td>
<td>Stretto inverso and recto (canon, incomplete statements)</td>
<td>iv-III-i; section ends on half cadence</td>
<td>52-53</td>
<td>5 (fourth)</td>
<td>4: a#-d#</td>
<td>iv/d# Phrygian</td>
<td>Stretto recto; canon; modified subject beginning on the second. g# minor implied; however, could also be interpreted as d# Phrygian, based on the melodic schema of the tonal answer. Series of strettoed tonal answers create stacks of harmonic (and melodic) fifths</td>
</tr>
<tr>
<td>52(b.2) -53</td>
<td></td>
<td></td>
<td>5</td>
<td>4: a#-d#</td>
<td>iv/d# Phrygian</td>
<td>Stretto recto</td>
<td></td>
</tr>
<tr>
<td>52(b.3) -54</td>
<td></td>
<td></td>
<td>5</td>
<td>4: a#-d#</td>
<td>iv/d# Phrygian</td>
<td>Stretto recto</td>
<td></td>
</tr>
<tr>
<td>54-55</td>
<td>10 (minor seventh)</td>
<td>5i: c#-f#</td>
<td>III (F#)</td>
<td>Stretto inverso (inverted subject). Series of strettoed inverted subject create cascades of melodic and harmonic fifths</td>
<td></td>
<td></td>
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<td>-------------------------------------------------</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>54(b.2)</td>
<td>10</td>
<td>5i: c#-f#</td>
<td>III (F#)</td>
<td>Stretto inverso (inverted subject)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>54(b.3)</td>
<td>10</td>
<td>5i: c#-f#</td>
<td>III (F#)</td>
<td>Stretto inverso (inverted subject)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>57(b.3)</td>
<td>0</td>
<td>4: e#-a# 4: d#-g#</td>
<td>i</td>
<td>Modified subject that starts on scale degree 2 instead of 1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>61-end</th>
<th>Augmentati on stretto</th>
<th>i-III-VI-I; deceptive cadence in B major (VI)</th>
</tr>
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<tbody>
<tr>
<td>61(b.3)</td>
<td>0</td>
<td>4: e#-a# 4: d#-g#</td>
</tr>
<tr>
<td>62-67</td>
<td>5 (fourth)</td>
<td>4: a#-d# 4: g#-c#</td>
</tr>
<tr>
<td>64(b.3)</td>
<td>5 (fourth)</td>
<td>5i: a#-d# 4i: g#-d#</td>
</tr>
<tr>
<td>67-69</td>
<td>3 (minor third)</td>
<td>4: g#-c# 4: f#-b</td>
</tr>
<tr>
<td>67(b.3)</td>
<td>3 (minor third)</td>
<td>4: g#-c# 4: f#-b</td>
</tr>
<tr>
<td>69(b.3)</td>
<td>8 (minor sixth)</td>
<td>4: c#-f# 4: b-e natural</td>
</tr>
<tr>
<td>72(b.3)</td>
<td>3 (minor third)</td>
<td>4: f#-b 4: g#-c#</td>
</tr>
<tr>
<td>77-79</td>
<td>0</td>
<td>4: e#-a# 4: d#-g#</td>
</tr>
<tr>
<td>77(b.2.5)-80</td>
<td>0</td>
<td>4: e#-a# 4: d#-g#</td>
</tr>
<tr>
<td>77(b.3)</td>
<td>0</td>
<td>4: e#-a# 4: d#-g#</td>
</tr>
<tr>
<td>80(b.3)</td>
<td>5</td>
<td>4: a#-d# 4: g#-c#</td>
</tr>
</tbody>
</table>
4 BROADER IMPLICATIONS AND SIGNIFICANCE OF STUDY

In a nutshell, what I seek to illuminate through this study is a dimension of Bach’s music—a heightened sense of color engendering an emotional affect that is inextricably tied to the subtle variation of interval sizes—that has been essentially forgotten in an era that has eradicated this diversity through the hegemony of equal temperament. What this also suggests is that there is far more import to tuning than meets the eye, and our modern system of seeing the tuning and tempering of the keyboard as a scientific and mechanical endeavor separate from artistic production is doing a great disservice to our sensitivity to tonality, harmony, and musical coloration. Historically, the integral role that tuning and temperament played in regards to its control over harmony and coloration was understood and appreciated by composers and theorists, and tuning was viewed as more of an art rather than a science. Furthermore, there was no dichotomy between the role of a tuner and the role of a musician—composers and performers were expected to be able to tune their own instruments, and although there were general guidelines to follow for the procedure, the subtle nuances were not codified, thus allowing for degrees of variation in tempering across different individuals, very much akin to variations that arise across different performers of the same piece due to artistic voice. Additionally, systems that featured different sized semitones—leading to wider array of intervals, which in turn govern key coloration—were favored over the uniformity and blandness of equal temperament.

Ultimately, the modern era with its dissociation of the tuner from the performer, as well as the widespread standardization of equal temperament to the point of the rendering unequal systems obsolete, has divested us of certain awareness and understanding of the full artistic implications that the composers intended. It is my hope that this dissertation can open up new avenues of analysis leading to a more complete understanding of Bach’s music, not just as a form of reenactment and rediscovery, but also as a platform for a more diversified way of looking at harmony and its creative agents, transforming our experience of music that we come in contact with and future compositions that we aspire to create.
BIBLIOGRAPHY:


_____ . Musiktheoretische Schriften: Harmonologia musica / Musikalische Paradoxal-Discourse Faksimile der Ausgaben Frankfurt/Leipzig 1702 und Quedlinburg 1707. Introductory essay (German and English) to the facsimile reprint, Laaber-Verlag, 2002.