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5. MEETING THE CHALLENGES OF MATHEMATICS TEACHER EDUCATION THROUGH DESIGN AND USE OF TASKS THAT FACILITATE TEACHER LEARNING

This chapter presents seven unifying themes of tasks used in mathematics teacher education. These themes reflect broad goals for mathematics teacher education, and are closely related to the knowledge required for teaching mathematics. The challenges that teacher-educators face in having to design and implement productive tasks for teachers are discussed and illustrated through a specific task. The iterative process of design of this task is attributed, on the one hand, to the teacher-educator’s reflection in and on the task implementation; on the other hand, to her professional growth that is expressed in her increased appreciation of and attention to the above central themes. In addition, the interplay between teacher-educators’ roles as: researchers, facilitators of teacher learning, and designers of tasks for teacher education are addressed. The chapter concludes with a framework for examining the demands on and expectations of teacher-educators as facilitators of teacher learning and the role of tasks in enhancing teacher learning.

TEACHER-EDUCATORS’ CHALLENGES

Conceptual Framework

Teacher education seeks to transform prospective and/or practicing teachers from neophyte possibly uncritical perspectives on teaching and learning to more knowledgeable, adaptable, analytic, insightful, observant, resourceful, reflective and confident professionals ready to address whatever challenges teaching mathematics presents. This view of teacher education presents a great challenge for teacher-educators, who are in charge of facilitating teacher learning towards these goals. In this chapter I focus specifically on the teacher-educator role as task designer.1

There is a consensus among mathematics educators that learning occurs through engagement in tasks (Krainer, 1993; Simon & Tzur, 2004; Zaslavsky, 2005). Hiebert & Wearne (1993) assert that what people learn “is largely defined by the tasks they are given” (p. 395). Along these lines, Kilpatrick, Swafford and Findell

1 Although there are many other roles that can be considered that do not necessarily involve explicit tasks (e.g., general professional mentoring and support).
ORIT ZASLAVSKY

(2001) claim that the quality of instruction depends to a large extent “on whether teachers select cognitively demanding tasks, plan the lesson by elaborating the mathematics that the students are to learn through those tasks, and allocate sufficient time for the students to engage in and spend time on the tasks” (ibid, p. 9). Sierpinska (2004) considers the design, analysis and empirical testing of tasks one of the major responsibilities of mathematics education.

Transforming this idea to teacher education, the underlying assumption is that tasks play a critical role in teacher learning, similar to their role in students learning. The above transformation occurs optimally through constructive engagement in tasks that foster knowledge for teaching mathematics. Ideally such tasks provide a bridge between theory and practice, and serve to challenge, surprise, disturb, confront, extend, or provoke examination of alternatives, drawn from the context of teaching. In this context, tasks are seen as problems or activities that, having been developed, evaluated and refined over time, are posed to teacher education participants. Such participants are expected to engage in these tasks collaboratively, energetically, and intellectually with an open mind and an orientation to future practice. These tasks might be similar to those used by classroom teachers or idiosyncratic to teacher education.

The demands on teacher-educators, in terms of knowledge and qualities, are enormous and multifaceted. An overarching demand is for teacher-educators to be reflective practitioners (Schön, 1987). They need to constantly reflect on-action and in-action, in all phases of their work (e.g., planning, interacting with teachers, observing teachers’ work). Based on Steinbring (1998), Figure 1 suggests a general recursive mechanism of mutual construction of knowledge of both facilitator and learners. Although Steinbring referred in his model to teacher-student learning, this model can be extended to teacher-educators as facilitators and teachers as learners; it explains how teacher-educators learn through their practice. Thus, it provides insight into the role of teacher-educators2 as designers and orchestrators of tasks that foster teacher learning, and at the same time highlights the dynamic nature of teacher-educators’ practice and development.

Mostly, teacher-educators are ‘self made’. They make their own transitions from their experiences as mathematics teachers and/or as researchers of mathematics learning and teaching. There are relatively few explicit curricula for mathematics teacher education, particularly for programs for practicing teachers (with some exceptions described by Even, this volume), nor agreed upon content that needs to be “covered”. Consequently, teacher-educators are free to make choices with minimal constraints, yet they need to be clear about the goals they attempt to address in their work with teachers. Examining some of the “big ideas” for teacher education and looking at the goals that are put forth for approaches to teacher education points to a very broad and sound knowledge base required of mathematics teacher-educators.

2 In this chapter the term ‘teacher-educator’ refers to mathematics teacher-educators that are responsible for the learning of prospective and/or practicing mathematics teachers.
In order to understand the role and responsibility of teacher-educators in orchestrating mathematics teacher learning and development, and discuss the knowledge base teacher-educators draw on and the kind of practice and qualities they need to develop, I propose to examine a number of unifying themes that reflect goals for mathematics teacher education. These interrelated and partly overlapping themes are not based on the conventional content topics of teacher education (e.g., teaching decimals, grouping practices); they concern qualities and kinds of competence and knowledge that mathematics teacher education seeks to promote in prospective and practising teachers in a broad sense.

The themes are:
1. Developing adaptability
2. Fostering awareness to similarities and differences
3. Coping with conflicts, dilemmas and problem situations
4. Learning from the study of practice
5. Selecting and using (appropriate) tools and resources for teaching
6. Identifying and overcoming barriers to students’ learning
7. Sharing and revealing self, peer, and student dispositions

I turn to a succinct description of the above themes, and the challenges with which they present teacher-educators. It should be noted that each theme can be regarded from both mathematical and pedagogical perspectives. Following the description of

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The central themes around which this chapter is built were developed jointly with Peter Sullivan in the course of structuring a forthcoming edited book, by Springer: Zaslavsky, O. & Sullivan, P. (Eds.), Constructing knowledge for teaching secondary mathematics: Tasks to enhance prospective and practicing teacher learning.
these themes, I illustrate how a teacher-educator may meet these challenges through carefully designed tasks.

Theme 1: Developing adaptability. A unifying theme in many aspects of teacher education is the development in teachers of an orientation to being adaptable, to considering variations to questions, tasks, and intended curriculum, to searching for alternatives to unsuccessful approaches, and to adapting existing resources to intended goals. This kind of orientation can be considered as adaptability and concurs with Cooney’s (1994) ideas of adaptation, as well as notion of contingency discussed by Rowland, Huckstep, & Thwaites (2005). Especially in a teaching and learning environment that encourages active learning by students, there is a need for teachers to be prepared to make active responses, and these cannot be planned in advance. Thus, adaptability is inter-related to flexibility (Leikin & Dinur, 2007). Indeed, it is not only a desirable orientation, but also a desirable personal quality. Teacher adaptability can be useful in diverse situations. Often teacher adaptability and flexibility are tightly connected to knowing to act in the moment (Mason, 1999).

In order to help teachers become adaptable a teacher-educator first must be adaptable and exhibit this quality in the course of working with teachers. Thus, it is important that the teacher-educator orchestrates situations in which he or she can model flexibility and the ability to vary and consider alternatives; moreover, it is also critical to provide experiences for teachers to engage in activities that require in the moment decisions that encourage flexibility and adaptability to unexpected situations.

Theme 2: Fostering awareness to similarities and differences. Noticing similarities and differences, in the broad sense, is at the heart of learning and teaching (Mason, 1998). It is well known that the gradual process of associating concepts with categories is a critical aspect of learning. Classification of different objects according to various criteria may enhance awareness of ways in which they are related to each other (Silver, 1979). This process requires the identification of similarities and differences between objects along several dimensions; in the course of classifying objects, objects that initially appear different, may be considered the same from a certain criterion. For example, when classifying numbers according to their parity, 2 and 3 are different; however, when using the criterion of “primeness”, 2 and 3 are the same. Awareness to similarities and differences between mathematical objects is considered fundamental to mathematical thinking; comparing and contrasting is needed also in order to identify patterns and make connections between and across topics, contexts, types of problems and even between teaching approaches. In the latter, pedagogical considerations are involved.

In order to help teachers develop a tendency to notice and an ability to identify similarities and differences as a state of mind, and particularly in classroom situations, the teacher-educator must engage teachers in activities that require this
Theme 3: Coping with conflicts, dilemmas and problem situations. Teachers constantly face dilemmas and need to make decisions and choices under conflicting constraints, and deal with uncertainty and complexity (Sullivan & Mousley, 2001; Sullivan, 2006); they need to be problem solvers in the broad sense of this term, and enhance their students’ ability to solve mathematical problems and cope with impasses and cognitive conflict. Thus, it is imperative that teachers are prepared for dealing with this complex terrain, both as teachers – that is, as designers and orchestrators of such situations for their students, and as learners – that is, as active problem solvers. Sánchez & García, this volume, describe and analyse the dilemmas teacher-educators face in their decision making processes, which to some extent mirror teachers’ dilemmas.

Examination of what it takes for a mathematics teacher-educator to deal with the challenges of this theme points to a very broad knowledge base and several personal qualities that are required. In addition to being a competent problem solver and familiar with the relevant content and pedagogy, a teacher-educator is expected to be confident enough to engage teachers in open ended problem situations to which the possible solutions and new questions that may arise are not necessarily known to him/her in advance. A teacher-educator must be open minded and willing to accept and explore in real time unexpected approaches and ideas that teachers may suggest. This is similar to the demands on teachers to exhibit the same approaches with their students.

Theme 4: Learning from the study of practice. Many teacher education programs are seeking ways to enhance the practical relevance of their curriculum, while allowing prospective teachers opportunities to review key theoretical perspectives, and ultimately to develop a career long orientation to learning from the study of their own teaching or the teaching of others. There have been several approaches to learning from the study of practice. These include the realistic simulations offered by videotaped study of exemplary lessons (Clarke, 2000); interactive study of recorded exemplars (e.g., Merseth & Lacey, 1993); case methods of teaching dilemmas that problematise aspects of teaching (e.g., Stein, Smith, Henningsen, & Silver, 2000); and Lesson Study that engages teachers in thinking about their long-term goals for students, developing a shared teaching-learning plan, encountering tasks that are intended for the students, and finally observing a lesson and jointly discussing and reflecting on it (e.g., Lewis, Perry, & Hurd, 2004; Fernandez & Yoshida, 2004). Each of these requires appropriate prompts to critical analysis to be effective. In each case, the teacher learning is through the opportunity to view and review exemplars, to discuss with peers interpretations of the exemplars, to engage in critical dialog on the experience, and to hear informed analysis of both the practice and the experience of critique.
Teacher-educators who design and implement these experiences are presented with great challenges. They need to be able to capture problematic and insightful classroom situations, and translate them into challenging cases for teachers to ponder. Fostering critical discussions regarding such cases requires high level metacognitive and mentoring skills.

Theme 5: Selecting and using (appropriate) tools and resources for teaching. Selecting appropriate tools for mathematics teaching and using them effectively is a major challenge for teachers. Tools can be text books, additional readings, manipulatives, construction and measuring devices, transparencies, graphical calculators, and other technological environments. Making educated choices regarding what tools to use for certain purposes and how to use them requires familiarity with a wide range of tools from both a learner’s and a teacher’s perspective. It also requires awareness of the potential and limitations of each tool, for various purposes and contexts, and confidence in using it for teaching.

Tools can be seen in their broadest sense, to include many different kinds of resources, including human and cultural resources, such as language and time. Adler (2000) argues that increasing attention should be given to resources in mathematics teacher education from two aspects:

First, mathematics teacher education programmes need to work with teachers to extend common-sense notions of resources beyond material objects and include human and cultural resources such as language and time as pivotal in school mathematical practice. Second, attention in professional development activities needs to shift from broadening a view of what such resources are to how resources function as an extension of the mathematics teacher in the teaching-learning process. (ibid, p. 207).

From a mathematics teacher-educator perspective, enhancing teachers’ competence in selecting and effectively using tools for teaching requires a familiarity with a wide range of available tools and appreciations of their potential for accomplishing various goals; it also requires great sensitivity to teachers’ reluctance to incorporate unfamiliar innovative tools in their teaching.

Theme 6: Identifying and overcoming barriers to students’ learning. Education and schooling strive to overcome some real, and in some cases substantial, barriers and create opportunities for all students, especially those who would not otherwise have those opportunities. One of the challenges for teacher education is to educate prospective and practising teachers on the existence and sources of barriers, and of strategies that can be effective in assisting students to overcome those barriers (Sullivan, Zevenbergen, & Mousley, 2003). The barriers are associated with diversity, and might be due to: epistemological aspects of mathematics (e.g., informal vs. formal approaches, modes of representation; missed prior learning opportunities; learning styles); cultural factors including community expectations, gender, school/home aspirational mismatches; language barriers and usage;
TEACHER-EDUCATORS AS TASK DESIGNERS

physical and other disabilities; socio-economic factors, and others (Trentacosta & Kenney, 1997).

A teacher-educator, who intends to address this theme, must be aware of such barriers not just for students learning but also for teacher learning. One way to enhance teachers’ awareness and appreciation of barriers to student learning is to experience overcoming of barriers to their own learning. To do this, a teacher-educator must understand the nature and causes of such barriers (e.g., mathematical, representational, communicational), and be familiar with possible productive interventions. He or she needs to be able to address any prejudices or knowledge mismatches within the prospective or practising teachers, and design experiences that can assist teachers in intervening effectively to overcome barriers for their own learning as well as for their students. These experiences should help teachers develop ways to engage all students, especially those who may sometimes feel alienated from mathematics and schooling, in meaningful mathematical thinking and learning.

Theme 7: Sharing and revealing self, peer, and student dispositions. In the multidimensional endeavour of teaching and learning mathematics, and learning to teach mathematics, a key dimension is the disposition of the (prospective and practicing) teacher as a learner, the teacher as a teacher, and the pupil as a learner. The dimension of disposition is itself multifaceted. It can include the following overlapping categories:

• Beliefs about: the nature of mathematics; the utility of mathematics; the way mathematics is learned; one’s own ability to learn mathematics;
• Self-regulatory behaviours such as: persistence; self-efficacy; motivation; resilience;
• Attitudes such as: liking for mathematics; enjoyment of mathematics; mathematics anxiety;

Indeed almost all aspects of teacher education have an attitudinal or dispositional dimension that should be considered. There is an agreement that helping teachers become aware of their own beliefs, as well as their students’ dispositions, is “a significant step toward improving students’ opportunities to learn mathematics” (Mewborn & Cross, 2007, p. 262).

It follows that a teacher-educator needs to know about the multifaceted dimension of beliefs and dispositions and their effects on various aspects of learning and teaching mathematics. Moreover, it is important for a mathematics teacher-educator to exhibit positive dispositions and enthusiasm towards mathematics and learning mathematics.

The above themes convey some of the challenges of mathematics teacher education. As mentioned earlier, tasks play a critical role in (teacher) learning, thus, in meeting these challenges. From a teacher-educator’s perspective, designing and using worthwhile tasks for prospective or practicing teachers is a non-trivial task. The following sections deal with the special role of teacher-educators as designers of tasks that foster mathematics teacher learning.
Where Do Tasks Come from?

One of the main differences between the role of a teacher-educator and of a teacher in selecting and designing appropriate tasks for enhancing learning is the availability of appropriate resources. While a teacher has many accessible textbooks, teacher guides, and enrichment material that are easily accessible, a teacher-educator has very few resources to draw on directly. First, many teacher-educators programs have broad goals (such as the ones described above) with no specific curriculum. It is often left to the teacher-educator how to address these goals, in terms of content and coverage. In addition, there are hardly any textbooks for mathematics teacher learning. Thus, the teacher-educator has much freedom and flexibility in the choice he or she makes, and at the same time has very little material to build on. The main sources that are specific for teacher-educators (that is, in addition to the resources available for teachers) are professional journals and books, research papers and other accounts of practitioners, and personal experience, all related rather indirectly to tasks for teacher education. Experienced teacher-educators have a state of mind of constantly searching for ideas that can be turned into productive tasks for teachers. Such ideas are often encountered through interactions with colleagues at professional conferences and other occasions.

Another source of ideas for tasks for teacher education is related to the interplay between research and practice. There have been numerous calls for connecting research to teacher practice (e.g., Jaworski & Gellert, 2003; Heid et al., 2006). Connecting research and the practice of teacher-educators lends itself well, since most teacher-educators are also active researchers, who tend to conduct long term research programmes, or at least integrate action research in their work (e.g., Jaworski, 2001, 2003). An example of such connection is the insightful account García, Sánchez, & Escudero (2006) provide, highlighting the interrelations between their roles as researchers and teacher-educators.

Often, research informs and enhances teacher-educators’ teaching and vice versa through tasks. Zaslavsky (2005) points to the dual purpose tasks serve for teacher-educators. On the one hand, tasks are the means and content by which teacher-educators may enhance teacher learning. On the other hand, through a reflective process of designing, implementing, and modifying tasks, they turn into means of the teacher-educator’s learning. This kind of learning can be seen as learning through a form of action research of the teacher-educator, who constantly researches his or own practice. However, research may provide a rich source of ideas for tasks for teachers in many other ways as well. Zaslavsky and Zodik (2007) developed tasks for teachers based on their study of teachers’ use of examples. Their classroom observations led them to realize some features of example choice and use of which they were not aware. They used their findings for eliciting teachers’ discussions surrounding problematic aspects of exemplification, based on real practice. Similarly, the work of Stein et al. (2000) has emerged from
authentic classroom observations, which were used as the basis for designing cases in the form of tasks for teacher education.

Thus, there is a fruitful interplay between teacher-educator’s roles as: researchers, facilitators of teacher learning, and designers of tasks for teacher education (Figure 2). This interplay is a driving force for effective learning of the entire community of mathematics educators, including mathematics teachers, teacher-educators, and educators of teacher-educators (see Goodchild, this volume).

In order to get a glimpse at what is involved in designing a productive task for mathematics teacher education, I turn to an example of a task that has evolved over years of experience, through an iterative reflective process, until reaching a stage where unexpected responses from participants hardly occur. My experience as teacher-educator shows that at the initial stages, tasks tend to undergo substantial changes as a result of experiencing them with teachers. However, after several iterations, with many different groups of teachers, the process stabilizes, and in most cases no further modifications seem to be needed. Through the following example I draw connections between the design of the task and the seven themes discussed above, and show how these themes are reflected in the process of task selection, design and modification. This evolution is influenced by ongoing reflection on the task implementation (as described in the model in Figure 1), but also by a gradual professional growth of the teacher-educator in which appreciation of the above themes develops beyond the scope of a specific task.

**The Spiral Task**

The following task (Figure 3, Figure 4) was inspired by Burke (1993). It is very similar to the original (ibid) and can be seen as a well structured and directed investigation.

As a teacher-educator searching for ideas for challenging mathematical tasks for prospective mathematics teachers, over a decade ago I came across this activity that dealt with a perplexing and surprising problem of the existence of a pair of triangles that are not congruent yet have 5 congruent parts/elements (that is a combination of angles and sides).
Figure 3. The initial version of the “Spiral Task”: Structured investigation - part 1 (based on Burke, 1993)

This task addresses a counter-intuitive case, since students and teachers tend to think, based on their experiences with the triangles congruency theorems, that if two triangles have more than 3 congruent elements, (particularly if they have 5), then the triangles are necessarily congruent. It seemed appropriate for prospective and practising mathematics teachers because it dealt with challenging mathematics, for both students and teachers; it lends itself to making connections to other areas of mathematics and to the real world. Indeed, the teachers were highly engaged in
TEACHER-EDUCATORS AS TASK DESIGNERS

NECESSARY AND SUFFICIENT CONDITIONS FOR CONSTRUCTING OTHER SPIRALS CONSISTING OF 5-CON TRIANGLE PAIRS

1. For each triplet of numbers, try to construct a triangle with side lengths corresponding to the given triplets:
   (a) 6, 18, 54; (b) 18, 54, 162; (c) 16, 24, 36; (d) 24, 36, 54. Which cases are possible and which not? What properties does each triplet have?

2. Construct the ("possible") triangles corresponding to triplets (c) and (d), cut them out, and compare their angles. What can you infer about these triangles? Are they 5-con triangles? Can you place them one next to the other so they form part of a spiral?

3. As you saw, all the above triplets form an increasing geometric sequence. For (a) and (b) no corresponding triangles exist, while for (c) and (d) there are corresponding triangles that exist. Moreover, the two corresponding triangles (one with side lengths of 16, 24, 36 and the other with side lengths 24, 36, 54) are 5-con triangle pairs and placed accordingly, can form part of a spiral. If we denote the elements of the sequences as: a, aq, aq², aq³, …, what is the difference between the triplets for which a corresponding triangle exists and those for which no corresponding triangle exists?

4. From your above investigation, it follows that a necessary condition for a geometric sequence of 3 elements to have a corresponding spiral is that: q>1, a+aq>aq², that is: 1+q>q². What conditions does this imply for q? Are these sufficient and/or necessary conditions for a spiral of 5-con triangle pairs?

5. What do you know about the number: \( \frac{1+\sqrt{5}}{2} \). How does it relate to this investigation?

Figure 4. The initial version of the “Spiral Task”: Structured investigation - part 2 (based on Burke, 1993)

this task, and most of them were genuinely surprised and felt they learned a lot from it, mostly mathematically. They also appreciated the structure of the task, that is, the way one question leads to another. In addition to the mathematical knowledge and insight that this task promoted, the task offered teachers a mathematical investigation that could be offered to their students in the same way.

As mentioned previously, not all tasks for teachers are necessarily similar to tasks for students. Through tasks that are (almost) equally as suitable for teachers as for students, teachers may gain a better appreciation of what is entailed in the task by actually coping with the activity as learners.
After a rather long period of time using this task successfully with prospective as well as practicing teachers, and based on my personal development as teacher-educator, which included an increasing appreciation for open ended tasks, I decided to modify the activity turning it into a more open task (in the spirit of Zaslavsky, 1995). I also was concerned with teachers’ reluctance to engage in and facilitate cooperative learning. This led to the second version of the Spiral Task (Figure 5).

**TRIANGLE-SPIRALS**

In the following drawing there are 4 triangles arranged in a spiral shape. You have the identical drawing on a transparency sheet. You are asked to investigate the special properties of this spiral, find connections between the triangles that constitute it, and finally continue the spiral by adding two triangles to it. Try to use the transparency sheet for comparing measures of the different elements of the triangles.

Look at your peers and compare: Did you continue the spiral in the same way? Consider the limitation of the accuracy of the drawing and your measuring tools: How might they affect your conjectures and conclusions? In your investigation you probably found that each pair of adjacent triangles has exactly 5 congruent elements, yet they are not congruent. Such triangles are called 5-con triangles. Can you construct other pairs of 5-con triangles that are not part of the given spiral?

*Figure 5. The second version of the “Spiral Task”*

In the second version, there is no example disclosing the target connections, thus, there was a “risk” that not all of these connections would be identified. Instead, there is an invitation to peer interactions and comparisons. The intention was to reinforce the participants’ awareness of the social aspect of learning in problem solving situations. Generally, the second version of the Spiral Task led to more expressions of surprise, excitement, and enthusiasm when used with teachers. It evoked genuine collaborative work, with full participation of all group members.
Note that there was also a change in the measuring tools that were suggested: instead of cutting out triangles and using them to compare angles and sides, in the second version there is a use of a transparency sheet. That is, attention was also given to tools. Clearly, there is a difference if one needs to measure actual lengths of sides and angle measures, or whether one needs to compare the measures of pairs of sides or angles, just to detect equal or non-equal measurements. For the latter, transparencies can be very useful. Since teachers’ use of transparencies up to then was mostly for presentations, I thought this would be an opportunity to draw teachers’ attention to other possible uses of transparencies. (As a matter of fact, at that time I designed other activities for teachers with the use of transparencies, which turned insightful not only in terms of the mathematics but also in terms of expanding their familiarity and appreciation of tools. These activities included explorations of the properties of inverse functions, as well as a qualitative approach to constructing the graph of the derivative function directly from the graph of a function).

Figure 6 illustrates the outcome of a group of teachers who rotated counter-clockwise the spiral (congruent to the given one) that was drawn on the transparency. This rotation made the comparison between adjacent triangles transparent and the connections obvious. Not only had the congruence of angles and sides become clear but also the similarity between each two adjacent triangles. To most teachers, this was a new way of using transparencies. It demonstrated a qualitative way of identifying patterns, without “numbers”. Since some teachers used other tools (e.g., a protractor and ruler) there was a natural opportunity to discuss the affordances and limitations of each tool.

![Figure 6. Comparing measurements with a transparency sheet in the second version of the “Spiral Task”](image)

Following the above experiences with teachers, I realized that this task could also serve for enhancing teachers’ use and appreciation of advanced technological tools, and could involve more uncertainty with respect to the existence of such
Triangle Spiral (of the kinds described by Zaslavsky, 2005). Hence, I tried to create a situation which would also allow teachers to experience some degree of frustration, followed by ways to eliminate their frustration, with the hope that this would be a springboard for gaining appreciation to some possible barriers to students’ learning. Thus, the third modification was done with these goals in mind (Figure 7).

Is it possible to construct (in a dynamic geometry environment or with a compass and edge) a spiral shape like the one sketched above, that satisfies the following conditions?

- OA = BC
- OB = CD
- OC = DE

If it is possible, construct a spiral of this kind. How many are there? If not – why is it not possible?

Figure 7. The third version of the "Spiral Task"

In the third version the questioned connections between the sides and angles of each pair of adjacent triangles are given, and the task is to find out if these can co-exist, and if so – under what conditions. Through the attempts to construct such a spiral, without knowing for sure whether or not it is at all possible, teachers were engaging in authentic exploration that arose from the need to answer the “big” question. Some used a DGE and others turned to a compass and edge. Similarly as in the second version, this created a natural opportunity to compare the advantages and limitations of each tool for this particular investigation.

Most teachers begin their attempts to construct the requested spiral with no prior analysis. Very soon they notice that there is more to this than meets the eye. Once they try constructing the spiral, they face sort of an impasse due to unexpected
results on the screen. Figure 8 depicts two types of impasse that usually occur in such cases: when trying to construct the second triangle, the two sides do not intersect at the required point (C in Figure 7). This impasse is similar to what Hadas, Hershkowitz, and Schwarz (2000) discuss. It leads some teachers to switch to the conjecture that it is actually impossible to construct a spiral satisfying the given conditions, and others, to turn to a deep mathematical analysis of the implications of the given constraints. Thus, some begin analyzing what the given connections imply in terms of the initial triangle (OAB). This analysis is triggered by an inner need, that is, by an impasse they face.

The given properties of the spiral imply that the initial triangle (and actually every triangle in the spiral) has a special property: the lengths of its sides form a geometric sequence, within certain restrictions. It is very unlikely that a triangle chosen at random will satisfy these conditions, thus it is very unlikely that someone will construct such a spiral without conducting this analysis. At the end, through social interactions, most groups manage to resolve the state of uncertainty, through which they encounter many surprising connections and insights (similar to those suggested by Movshovitz-Hadar, 1988). A film documenting the underlying processes of teachers and students dealing with the Spiral Task highlights the similarities and differences between the two populations with respect to the uncertainty they experienced and the understandings they reached (I.F.A., 1997). This film serves for teacher workshops in which they relate to possible implementation of the task in their classrooms.

As mentioned above, this version evoked much uncertainty. It was implemented with many groups of prospective and practicing teachers who in turn tried it out with their students. To the vast majority of teachers and students the Spiral Task looked rather straightforward at first and the construction seemed feasible.

It should be noted that in all three versions, before handing out the task, teachers were prompted for their notions about the maximum number of pairs of congruent elements two non-congruent triangles could have. My experience is that in most groups there is a unanimous agreement that 3 is the maximum number, and that not just any 3 such pairs can exist in two non-congruent triangles. Few think that 4 could be, but all are convinced that if there are 5 pairs of congruent elements in two
triangles, the triangles are necessarily congruent. By this prompt, the surprise later on becomes more dramatic. Not only that they find out at the end that non-congruent triangles with 5 congruent elements exist, but they realize that there are an infinite number of such cases, and they even learn how to construct many different cases. This approach aims at addressing some beliefs teachers and students hold with respect to mathematics; it demonstrates that mathematics is stimulating and can be exciting.

Reflection on the processes teachers undergo while dealing with this version of the task leads to the distinction between the “machine’s” feedback and that provided by the facilitator. In this case, realizing that an arbitrary choice of sides and angles may not form a triangle, through observation of the outcome on the screen, is a manifestation that something “went wrong”, and this in itself motivates a further investigation or change of strategy, with no interference of the facilitator. This situation is an incentive for the participants to turn to a mathematical analysis of the given conditions. Peer discussions and debate evolve naturally, involving sound reasoning including questions of inferences, such as, what can be inferred if it appears not to work. Does it mean that such a spiral does not exist or does it imply that other approaches need to be applied before any conclusion can be reached?

Having gained experience with the three versions of the Spiral Task, I became more aware of the significance of varying the task, and began using all three with various groups of teachers. This was an opportunity to use the task for enhancing adaptability and dealing with task variation. After coping with one version (different participants begin with different versions), teachers moved to the other two and compared the three along several dimensions, such as challenge, openness, motivation and disposition, mathematical insights, learning opportunities and student diversity (with respect to the suitability of the different versions to different students). Teachers were also asked to suggest other versions for their own classroom. One offered to try out a lesson based on the Spiral Task with her 9th grade students, and invited other teachers to observe. Her lesson was documented and used for discussions focusing on the way tasks unfold in the classroom, and how similar or different the task unfolded in a teacher workshop compared to her classroom.

What Is the Spiral Task an Example of?

I use the Spiral Task to illustrate several issues. First, it conveys the challenges involved in designing worthwhile learning situations for prospective and practising mathematics teachers. These kinds of activities are not readily available for teacher-educators to offer. It is awareness of the teacher-educator to all the earlier discussed themes and goals for mathematics teacher education that may lead him or her to an ongoing process of addressing as many aspects of those themes in the actual work with teachers.

The Spiral Task also illustrates how a task may evolve in a way that addresses to some extent all seven of the themes presented earlier. It involved reflection and
adaptability on the part of the teacher-educator, in addition to other personal traits, that were modelled and discussed. It provided opportunities for teachers to engage in challenging problem solving and exploration, within school mathematics, coping with uncertainty and conflict, and required making comparisons and noticing connections. Each version of the task dealt to some extent with tools and their affordances. In the course of working with the third version teachers experienced as learners a certain degree of frustration, followed by ways of overcoming it, and reflected on the implications of these experiences for teaching. The task as a whole became a means for developing motivation and positive dispositions toward mathematical explorations.

Table 1 gives an overall picture of some of the ways in which each version addresses the seven themes, and conveys the added value of each version in terms of these themes. Movement from left to right in most rows indicates an increase in the amount of ways in which the themes are addressed. Moreover, the specific aspects addressed become more subtle and sophisticated.

Table 1. Some affordances of each version of the Spiral Task in terms of the seven themes for teacher education

<table>
<thead>
<tr>
<th>Versions</th>
<th>Themes</th>
<th>Initial Version</th>
<th>Second Version</th>
<th>Third Version</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Adaptability &amp; flexibility</td>
<td>Individual guided Investigation of the properties of a given spiral</td>
<td>Collaborative open investigation of the properties of different spirals</td>
<td>Collaborative open investigation of the conditions for the existence of a spiral with given properties</td>
<td>Teachers were given various versions, and considered the possible use of the alternative approaches with their students.</td>
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<td></td>
<td>Awareness to similarities, differences (incl. making connections &amp; identifying patterns)</td>
<td>Comparison between different pairs of adjacent triangles; Connection to similar triangles, Fibonacci numbers, golden section, real world phenomena.</td>
<td>In addition to what the initial version triggered: Comparison between different spirals – each group explored a different one;</td>
<td>Search for necessary and sufficient conditions for this kind of spiral; Classification of triangles according to their possible inclusion in such spirals;</td>
<td>The three task versions provided opportunities for comparing them along several dimensions, including pedagogical &amp; epistemological.</td>
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<tr>
<td>Versions</td>
<td>Initial Version</td>
<td>Second Version</td>
<td>Third Version</td>
<td>Comments</td>
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<tr>
<td>Themes</td>
<td>Individual guided Investigation of the properties of a given spiral</td>
<td>Collaborative open investigation of the properties of different spirals</td>
<td>Collaborative open investigation of the conditions for the existence of a spiral with given properties</td>
<td>Teachers’ notions were elicited in advance, regarding the maximum number of pairs of congruent elements of two non-congruent triangles.</td>
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<tr>
<td>Coping with</td>
<td>A counter-intuitive outcome; Challenging mathematics; Surprising results</td>
<td>Overcoming the conflict between intuition and evidence; Awareness to social</td>
<td>Coping with uncertainty; Subtle logical inferences; Need for careful analysis in order to deal with the impasse.</td>
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<td>conflicts,</td>
<td></td>
<td>aspects of open-ended problem solving situations.</td>
<td></td>
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<td>dilemmas, &amp;</td>
<td></td>
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<td></td>
<td></td>
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<td>problem situations</td>
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<tr>
<td>Learning from</td>
<td>The nature of the task and its structure allowed readily use with students.</td>
<td>One teacher invited her peers to learn from observing her use this version in</td>
<td>This version appealed to teachers who felt comfortable with DGE, and tried it out with their students.</td>
<td>Teachers tried out the different versions of the task with their students and shared their experiences with their peers.</td>
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<td>practice</td>
<td></td>
<td>her classroom.</td>
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<tr>
<td>Judicious use of</td>
<td>Possible use of ruler and protractor; Use of cut-out triangles for comparing side</td>
<td>Use of transparency for comparing side lengths and angle measurements;</td>
<td>Use of ruler and compass, vs. DGE for constructions and comparisons; Non-judgmental feedback provided by a technological tool – by gap between expected vs. actual outcome.</td>
<td>Experiencing all three versions and reflecting on the different tools employed, created a powerful opportunity to developing appreciation of tools.</td>
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<tr>
<td>tools</td>
<td>lengths and angle measurements.</td>
<td>Appreciation of the limitations of measuring tools.</td>
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<tr>
<td>Overcoming</td>
<td>Overcoming uneasiness regarding open-ended investigation; Appreciation of inner</td>
<td>Overcoming impasses and dealing with frustration; Sense of joy in overcoming</td>
<td>Reflection on personal and group encounters; Discussing barriers, motivation, and inclusion, associated with learners.</td>
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<tr>
<td>barriers</td>
<td></td>
<td>difficulties.</td>
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For example, looking at the theme *Judicious use of tools*, the initial version is based on simple measuring tools (ruler and protractor); the second version offers a special use of transparency for comparing measurements and making connections transparent; the third version lends itself to technology – in which although measurements can be instantly displayed, the formulation of the task leads to better use of technology, by conveying the idea that the tool cannot replace a mathematical analysis. Instead, it can offer accurate drawings which form powerful feedback, particularly when there is a mismatch between what is expected and what actually happens.

It should be noted that not surprisingly, this one task, with its three versions, does not address all aspects of each theme. It would take many more carefully designed tasks, to capture the multifaceted goals and themes described earlier. However, as illustrated by the Spiral Task, keeping these multifaceted goals and themes in mind, could lead to gradual increase in the affordances of tasks with respect to meeting these goals and challenges.

**THE DEMANDS ON TEACHER-EDUCATORS AS FACILITATORS OF TEACHER LEARNING**

As reflected in the previous sections, the demands on mathematics teacher-educators are heavy, in terms of their knowledge-base, personal traits, and responsibilities. There are several personal traits that are considered desirable and even imperative for mathematics educators (teachers, teacher-educators and educators of teacher-educators). For example, a mathematics educator needs to be reflective, adaptive, flexible, open minded, risk taker, sensitive, confident, and enthusiastic about what he or she teaches. Some of these traits may be naturally developed over years of experience; however, tasks for teacher education can be seen as springboards for enhancing such personal traits as well. A common way of doing so is by modelling these traits in the course of using tasks. Yet, this is not enough. The implicit exhibition of such traits should be made explicit. That is, if a teacher-educator exhibits flexibility and is open to an unexpected idea, it is
important to come back to that at another stage and reflect on the kind of flexibility that was exhibited and what teachers gained from it as learners and what they can take from it as teachers.

Zaslavsky (2007) conceptualizes the enormous demands on teacher-educators, based on a collection of articles dealing with the nature and role of tasks in mathematics teacher education (Zaslavsky, Watson & Mason, 2007). Accordingly, the knowledge base of teacher-educators consists of the knowledge for teaching mathematics (that is, mathematics teachers’ knowledge base) as well as knowledge of how to enhance teacher learning. This could include the model offered by Zaslavsky & Leikin (2004) for discussing mathematics teacher-educators’ knowledge base, but also includes theoretical and practical knowledge of the seven central themes discussed earlier. In addition, teacher-educators are expected to hold and exhibit several personal traits that are needed for facilitating learning. Based on their knowledge and personal traits they are expected to engage teachers in productive tasks, during which they may model certain behaviours and traits, and later reflect on them and make them an explicit focal topic for discussion. The ultimate goal of this aspect of mathematics teacher-educators’ practice is to help teachers construct knowledge for teaching mathematics and develop personal traits needed for becoming a competent and effective teacher.

When discussing tasks, in general, and tasks for teacher education, in particular, it is hard to make a clear distinction between the actual task and how it unfolds in the learning setting. Tasks for teachers have several layers. As described above, and unlike tasks for students, a mathematical task for teachers rarely deals with just the mathematics. It can be seen as an opportunity to generalize from it to a large class of tasks, and to deal with many other aspects of teaching mathematics as well. Moreover, as Zsalavsky (2007) points out, “tasks evolve over years of reflective practice” of mathematics teacher-educators.

This chapter highlights the complexity and challenges of one (although central) aspect of mathematics teacher-educators’ roles and responsibilities. It conveys the high demands, and raises the question of how to prepare teacher-educators for these roles. There are relatively few programmes for developing mathematics teacher-educators (e.g., Even, 1999), and none address all the above themes in a substantial way. Learning from their own practice is certainly a way to develop (e.g., Zaslavsky & Leikin, 2004; Tzur, 2001), yet – it would be valuable to begin articulating the goals for mathematics teacher-educators’ education, and ways of achieving them.

REFERENCES


TEACHER-EDUCATORS AS TASK DESIGNERS


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